NAG Toolbox

nag_lapack_zgelss (f08kn)

1 Purpose

nag_lapack_zgelss (f08kn) computes the minimum norm solution to a complex linear least squares problem

$$\min_x \|b - Ax\|_2.$$

2 Syntax

```
[a, b, s, rank, info] = nag_lapack_zgelss(a, b, rcond, 'm', m, 'n', n, 'nrhs_p',
nrhs_p)
```

```
[a, b, s, rank, info] = f08kn(a, b, rcond, 'm', m, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_zgelss (f08kn) uses the singular value decomposition (SVD) of A, where A is an m by n matrix which may be rank-deficient.

Several right-hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m by r right-hand side matrix B and the n by r solution matrix X.

The effective rank of A is determined by treating as zero those singular values which are less than **rcond** times the largest singular value.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The m by n matrix A.

2: **b**(ldb, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The m by r right-hand side matrix B.

3: **rcond** – REAL (KIND=nag_wp)

Used to determine the effective rank of A. Singular values $\mathbf{s}(i) \leq \mathbf{rcond} \times \mathbf{s}(1)$ are treated as zero. If $\mathbf{rcond} < 0$, ***machine precision*** is used instead.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrices B and X.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: $\mathbf{a}(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{m})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

The first $\min(m,n)$ rows of A are overwritten with its right singular vectors, stored row-wise.

2: $\mathbf{b}(ldb,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1,\mathbf{m},\mathbf{n})$.

The second dimension of the array **b** will be $\max(1,\mathbf{nrhs\_p})$.

b stores the n by r solution matrix X. If $m \geq n$ and $\mathbf{rank} = n$, the residual sum of squares for the solution in the ith column is given by the sum of squares of the modulus of elements $n+1,\ldots,m$ in that column.

3: $\mathbf{s}(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **s** will be $\max(1,\min(\mathbf{m},\mathbf{n}))$

The singular values of A in decreasing order.

4: **rank** – INTEGER

The effective rank of A, i.e., the number of singular values which are greater than $\mathbf{rcond} \times \mathbf{s}(1)$.

5: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **s**, 9: **rcond**, 10: **rank**, 11: **work**, 12: **lwork**, 13: **rwork**, 14: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

The algorithm for computing the SVD failed to converge; if **info** $= i$, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

7 Accuracy

See Section 4.5 of Anderson *et al.* (1999) for details.

8 Further Comments

The real analogue of this function is nag_lapack_dgelss (f08ka).

9 Example

This example solves the linear least squares problem

$$\min_x \|b - Ax\|_2$$

for the solution, x, of minimum norm, where

$$A = \begin{pmatrix} 0.47 - 0.34i & -0.40 + 0.54i & 0.60 + 0.01i & 0.80 - 1.02i \\ -0.32 - 0.23i & -0.05 + 0.20i & -0.26 - 0.44i & -0.43 + 0.17i \\ 0.35 - 0.60i & -0.52 - 0.34i & 0.87 - 0.11i & -0.34 - 0.09i \\ 0.89 + 0.71i & -0.45 - 0.45i & -0.02 - 0.57i & 1.14 - 0.78i \\ -0.19 + 0.06i & 0.11 - 0.85i & 1.44 + 0.80i & 0.07 + 1.14i \end{pmatrix}$$

and

$$b = \begin{pmatrix} -1.08 - 2.59i \\ -2.61 - 1.49i \\ 3.13 - 3.61i \\ 7.33 - 8.01i \\ 9.12 + 7.63i \end{pmatrix}.$$

A tolerance of 0.01 is used to determine the effective rank of A.

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

```
function f08kn_example

fprintf('f08kn example results\n\n');

% Least Squares solution of Ax = b, where
a = [ 0.47 - 0.34i, -0.40 + 0.54i,  0.60 + 0.01i,  0.80 - 1.02i;
     -0.32 - 0.23i, -0.05 + 0.20i, -0.26 - 0.44i, -0.43 + 0.17i;
      0.35 - 0.60i, -0.52 - 0.34i,  0.87 - 0.11i, -0.34 - 0.09i;
      0.89 + 0.71i, -0.45 - 0.45i, -0.02 - 0.57i,  1.14 - 0.78i;
```

```
     -0.19 + 0.06i,  0.11 - 0.85i,  1.44 + 0.80i,  0.07 + 1.14i];
b = [-1.08 - 2.59i;
     -2.61 - 1.49i;
      3.13 - 3.61i;
      7.33 - 8.01i;
      9.12 + 7.63i];
[m,n] = size(a);

% treat singular values < 0.01 as zero
rcond = 0.01;
[~, x, s, rank, info] = f08kn( ...
                               a, b, rcond);

disp('Least squares solution');
disp(x(1:n));
disp('Tolerance used to estimate the rank of A');
fprintf('%12.2e\n',rcond);
disp('Estimated rank of A');
fprintf('%5d\n\n',rank);
disp('Singular values of A');
disp(s');
```

9.2 Program Results

```
     f08kn example results

Least squares solution
   1.1673 - 3.3222i
   1.3480 + 5.5028i
   4.1762 + 2.3434i
   0.6465 + 0.0105i

Tolerance used to estimate the rank of A
    1.00e-02
Estimated rank of A
    3

Singular values of A
    2.9979    1.9983    1.0044    0.0064
```