NAG Toolbox

nag_lapack_zstein (f08jx)

1 Purpose

nag_lapack_zstein (f08jx) computes the eigenvectors of a real symmetric tridiagonal matrix corresponding to specified eigenvalues, by inverse iteration, storing the eigenvectors in a complex array.

2 Syntax

```
[z, ifailv, info] = nag_lapack_zstein(d, e, m, w, iblock, isplit, 'n', n)
[z, ifailv, info] = f08jx(d, e, m, w, iblock, isplit, 'n', n)
```

3 Description

nag_lapack_zstein (f08jx) computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, by inverse iteration (see Jessup and Ipsen (1992)). It is designed to be used in particular after the specified eigenvalues have been computed by nag_lapack_dstebz (f08jj) with **order** = 'B', but may also be used when the eigenvalues have been computed by other functions in Chapters F02 or F08.

The eigenvectors of T are real, but are stored by this function in a complex array. If T has been formed by reduction of a full complex Hermitian matrix A to tridiagonal form, then eigenvectors of T may be transformed to (complex) eigenvectors of A by a call to nag_lapack_zunmtr (f08fu) or nag_lapack_zupmtr (f08gu).

nag_lapack_dstebz (f08jj) determines whether the matrix T splits into block diagonal form:

$$T = \begin{pmatrix} T_1 & & & & \\ & T_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & T_p \end{pmatrix}$$

and passes details of the block structure to this function in the arrays **iblock** and **isplit**. This function can then take advantage of the block structure by performing inverse iteration on each block T_i separately, which is more efficient than using the whole matrix.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Jessup E and Ipsen I C F (1992) Improving the accuracy of inverse iteration *SIAM J. Sci. Statist. Comput.* **13** 550–572

5 Parameters

5.1 Compulsory Input Parameters

1: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** must be at least $\max(1, \mathbf{n})$

The diagonal elements of the tridiagonal matrix T.

2: **e**(:) – REAL (KIND=nag_wp) array

The dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$

The off-diagonal elements of the tridiagonal matrix T.

3: **m** – INTEGER

m, the number of eigenvectors to be returned.

Constraint: $0 \leq \mathbf{m} \leq \mathbf{n}$.

4: **w**(:) – REAL (KIND=nag_wp) array

The dimension of the array **w** must be at least $\max(1, \mathbf{n})$

The eigenvalues of the tridiagonal matrix T stored in $\mathbf{w}(1)$ to $\mathbf{w}(m)$, as returned by nag_lapack_dstebz (f08jj) with **order** = 'B'. Eigenvalues associated with the first sub-matrix must be supplied first, in nondecreasing order; then those associated with the second sub-matrix, again in nondecreasing order; and so on.

Constraint: if $\mathbf{iblock}(i) = \mathbf{iblock}(i+1)$, $\mathbf{w}(i) \leq \mathbf{w}(i+1)$, for $i = 1, 2, \ldots, \mathbf{m} - 1$.

5: **iblock**(:) – INTEGER array

The dimension of the array **iblock** must be at least $\max(1, \mathbf{n})$

The first m elements must contain the sub-matrix indices associated with the specified eigenvalues, as returned by nag_lapack_dstebz (f08jj) with **order** = 'B'. If the eigenvalues were not computed by nag_lapack_dstebz (f08jj) with **order** = 'B', set $\mathbf{iblock}(i)$ to 1, for $i = 1, 2, \ldots, m$.

Constraint: $\mathbf{iblock}(i) \leq \mathbf{iblock}(i+1)$, for $i = 1, 2, \ldots, \mathbf{m} - 1$.

6: **isplit**(:) – INTEGER array

The dimension of the array **isplit** must be at least $\max(1, \mathbf{n})$

The points at which T breaks up into sub-matrices, as returned by nag_lapack_dstebz (f08jj) with **order** = 'B'. If the eigenvalues were not computed by nag_lapack_dstebz (f08jj) with **order** = 'B', set $\mathbf{isplit}(1)$ to **n**.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **d** and the second dimension of the array **d**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix T.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **z**(ldz, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **z** will be $\max(1, \mathbf{n})$.

The second dimension of the array **z** will be $\max(1, \mathbf{m})$.

The m eigenvectors, stored as columns of Z; the ith column corresponds to the ith specified eigenvalue, unless **info** > 0 (in which case see Section 6).

2: **ifailv**(**m**) – INTEGER array

If $\mathbf{info} = i > 0$, the first i elements of **ifailv** contain the indices of any eigenvectors which have failed to converge. The rest of the first **m** elements of **ifailv** are set to 0.

3: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **d**, 3: **e**, 4: **m**, 5: **w**, 6: **iblock**, 7: **isplit**, 8: **z**, 9: **ldz**, 10: **work**, 11: **iwork**, 12: **ifailv**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0 (*warning*)

If **info** $= i$, then i eigenvectors (as indicated by the argument **ifailv** above) each failed to converge in five iterations. The current iterate after five iterations is stored in the corresponding column of **z**.

7 Accuracy

Each computed eigenvector z_i is the exact eigenvector of a nearby matrix $A + E_i$, such that

$$\|E_i\| = O(\epsilon)\|A\|,$$

where ϵ is the ***machine precision***. Hence the residual is small:

$$\|Az_i - \lambda_i z_i\| = O(\epsilon)\|A\|.$$

However, a set of eigenvectors computed by this function may not be orthogonal to so high a degree of accuracy as those computed by nag_lapack_zsteqr (f08js).

8 Further Comments

The real analogue of this function is nag_lapack_dstein (f08jk).

9 Example

See Section 10 in nag_lapack_zunmtr (f08fu).

9.1 Program Text

```
function f08jx_example

fprintf('f08jx example results\n\n');

% Lower triangular part of Hermitian matrix A
uplo = 'L';
a = [-2.28 + 0.00i,  0.00 + 0i,     0    + 0i,     0    + 0i;
      1.78 + 2.03i, -1.12 + 0i,     0    + 0i,     0    + 0i;
      2.26 - 0.10i,  0.01 - 0.43i, -0.37 + 0i,     0    + 0i;
     -0.12 - 2.53i, -1.07 - 0.86i,  2.31 + 0.92i, -0.73 + 0i];

% Reduce A to tridiagonal form
[aq, d, e, tau, info] = f08fs( ...
                              uplo, a);

% Calculate two smallest eigenvalues
range = 'Indices';
order = 'Block';
vl = 0;
```

```
vu = 0;
il = nag_int(1);
iu = nag_int(2);
abstol = 0;
[m, nsplit, w, iblock, isplit, info] = ...
  f08jj( ...
         range, order, vl, vu, il, iu, abstol, d, e);

% Corresponding eigenvectors of T
[tz, ifailv, info] = f08jx( ...
                            d, e, m, w, iblock, isplit);

% TRansform to eigenvalues of A (by premultiplying by Q)
trans = 'No transpose';
side = 'Left';
[z, info] = f08fu( ...
                   side, uplo, trans, aq, tau, tz);

% Normalize vectors, largest element is real and positive.
for i = 1:m
  [~,k] = max(abs(real(z(:,i)))+abs(imag(z(:,i))));
  z(:,i) = z(:,i)*conj(z(k,i))/abs(z(k,i));
end

disp(' Selected eigenvalues of A:');
disp(w(1:m));
disp(' Corresponding eigenvectors:');
disp(z);
```

9.2 Program Results

```
     f08jx example results

 Selected eigenvalues of A:
   -6.0002
   -3.0030

 Corresponding eigenvectors:
   0.7299 + 0.0000i  -0.2120 + 0.1497i
  -0.1663 - 0.2061i   0.7307 + 0.0000i
  -0.4165 - 0.1417i  -0.3291 + 0.0479i
   0.1743 + 0.4162i   0.5200 + 0.1329i
```