NAG Toolbox

nag_lapack_zhbtrd (f08hs)

1 Purpose

nag_lapack_zhbtrd (f08hs) reduces a complex Hermitian band matrix to tridiagonal form.

2 Syntax

```
[ab, d, e, q, info] = nag_lapack_zhbtrd(vect, uplo, kd, ab, q, 'n', n)
[ab, d, e, q, info] = f08hs(vect, uplo, kd, ab, q, 'n', n)
```

3 Description

nag_lapack_zhbtrd (f08hs) reduces a Hermitian band matrix A to real symmetric tridiagonal form T by a unitary similarity transformation:

$$T = Q^{\mathrm{H}} AQ.$$

The unitary matrix Q is determined as a product of Givens rotation matrices, and may be formed explicitly by the function if required.

The function uses a vectorizable form of the reduction, due to Kaufman (1984).

4 References

Kaufman L (1984) Banded eigenvalue solvers on vector machines *ACM Trans. Math. Software* **10** 73–86

Parlett B N (1998) *The Symmetric Eigenvalue Problem* SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **vect** – CHARACTER(1)

Indicates whether Q is to be returned.

vect = 'V'
Q is returned.

vect = 'U'
Q is updated (and the array **q** must contain a matrix on entry).

vect = 'N'
Q is not required.

Constraint: **vect** = 'V', 'U' or 'N'.

2: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'
The upper triangular part of A is stored.

uplo = 'L'
The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **kd** – INTEGER

If **uplo** = 'U', the number of superdiagonals, k_d, of the matrix A.

If **uplo** = 'L', the number of subdiagonals, k_d, of the matrix A.

Constraint: $\mathbf{kd} \geq 0$.

4: $\mathbf{ab}(ldab,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **ab** must be at least $\max(1, \mathbf{kd}+1)$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The upper or lower triangle of the n by n Hermitian band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

5: $\mathbf{q}(ldq,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension, ldq, of the array **q** must satisfy

if **vect** = 'V' or 'U', $ldq \geq \max(1, \mathbf{n})$;
if **vect** = 'N', $ldq \geq 1$.

The second dimension of the array **q** must be at least $\max(1, \mathbf{n})$ if **vect** = 'V' or 'U' and at least 1 if **vect** = 'N'.

If **vect** = 'U', **q** must contain the matrix formed in a previous stage of the reduction (for example, the reduction of a banded Hermitian-definite generalized eigenproblem); otherwise **q** need not be set.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **ab** and the second dimension of the array **ab**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: $\mathbf{ab}(ldab,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **ab** will be $\max(1, \mathbf{kd}+1)$.

The second dimension of the array **ab** will be $\max(1, \mathbf{n})$.

ab stores values generated during the reduction to tridiagonal form.

The first superdiagonal or subdiagonal and the diagonal of the tridiagonal matrix T are returned in **ab** using the same storage format as described above.

2: **d**(**n**) – REAL (KIND=nag_wp) array

The diagonal elements of the tridiagonal matrix T.

3: **e**(**n** – **1**) – REAL (KIND=nag_wp) array

The off-diagonal elements of the tridiagonal matrix T.

4: **q**(ldq, :) – COMPLEX (KIND=nag_wp) array

The first dimension, ldq, of the array **q** will be

if **vect** = 'V' or 'U', $ldq = \max(1, \mathbf{n})$;
if **vect** = 'N', $ldq = 1$.

The second dimension of the array **q** will be $\max(1, \mathbf{n})$ if **vect** = 'V' or 'U' and at least 1 if **vect** = 'N'.

If **vect** = 'V' or 'U', the n by n matrix Q.

If **vect** = 'N', **q** is not referenced.

5: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **vect**, 2: **uplo**, 3: **n**, 4: **kd**, 5: **ab**, 6: **ldab**, 7: **d**, 8: **e**, 9: **q**, 10: **ldq**, 11: **work**, 12: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A + E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

The computed matrix Q differs from an exactly unitary matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $20n^2k$ if **vect** = 'N' with $10n^3(k-1)/k$ additional operations if **vect** = 'V'.

The real analogue of this function is nag_lapack_dsbtrd (f08he).

9 Example

This example computes all the eigenvalues and eigenvectors of the matrix A, where

$$A = \begin{pmatrix} -3.13 + 0.00i & 1.94 - 2.10i & -3.40 + 0.25i & 0.00 + 0.00i \\ 1.94 + 2.10i & -1.91 + 0.00i & -0.82 - 0.89i & -0.67 + 0.34i \\ -3.40 - 0.25i & -0.82 + 0.89i & -2.87 + 0.00i & -2.10 - 0.16i \\ 0.00 + 0.00i & -0.67 - 0.34i & -2.10 + 0.16i & 0.50 + 0.00i \end{pmatrix}.$$

Here A is Hermitian and is treated as a band matrix. The program first calls nag_lapack_zhbtrd (f08hs) to reduce A to tridiagonal form T, and to form the unitary matrix Q; the results are then passed to nag_lapack_zsteqr (f08js) which computes the eigenvalues and eigenvectors of A.

9.1 Program Text

```
function f08hs_example

fprintf('f08hs example results\n\n');

% Hermitian band matrix A, stored on symmetric banded format
uplo = 'L';
kd = nag_int(2);
n  = 4;
ab = [-3.13 + 0i,     -1.91 + 0i,    -2.87 + 0i,     0.5 + 0i;
       1.94 + 2.10i,  -0.82 + 0.89i, -2.10 + 0.16i,  0   + 0i;
      -3.40 - 0.25i,  -0.67 - 0.34i,  0    + 0i,     0   + 0i];

% Reduce A to tridiagonal form and compute Q
vect = 'V';
q = complex(zeros(n, n));
[abf, d, e, q, info] = f08hs( ...
                              vect, uplo, kd, ab, q);

% Calculate eigenvalues/vectors of A from Q, d and e.
compz = 'V';
[w, ~, z, info] = f08js( ...
                         compz, d, e, q);

% Normalize: largest elements are real
for i = 1:n
  [~,k] = max(abs(real(z(:,i)))+abs(imag(z(:,i))));
  z(:,i) = z(:,i)*conj(z(k,i))/abs(z(k,i));
end

disp('Eigenvalues');
disp(w');
[ifail] = x04da( ...
                 'General', ' ', z, 'Eigenvectors');
```

9.2 Program Results

```
f08hs example results

Eigenvalues
   -7.0042   -4.0038    0.5968    3.0012

 Eigenvectors
          1       2       3       4
 1   0.7293 -0.2128 -0.3354  0.4732
     0.0000  0.1511 -0.1604  0.1947

 2  -0.1654  0.7316 -0.2804  0.0891
    -0.2046  0.0000 -0.3413  0.4387
```

```
 3   0.6081  0.3910 -0.0144 -0.5172
     0.0301 -0.3843  0.1532 -0.1938

 4   0.1653  0.2775  0.8019  0.4824
    -0.0303 -0.1378  0.0000  0.0000
```