NAG Toolbox

nag_lapack_dsbtrd (f08he)

1 Purpose

nag_lapack_dsbtrd (f08he) reduces a real symmetric band matrix to tridiagonal form.

2 Syntax

```
[ab, d, e, q, info] = nag_lapack_dsbtrd(vect, uplo, kd, ab, q, 'n', n)
[ab, d, e, q, info] = f08he(vect, uplo, kd, ab, q, 'n', n)
```

3 Description

nag_lapack_dsbtrd (f08he) reduces a symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation:

$$T = Q^{\mathrm{T}} AQ.$$

The orthogonal matrix Q is determined as a product of Givens rotation matrices, and may be formed explicitly by the function if required.

The function uses a vectorizable form of the reduction, due to Kaufman (1984).

4 References

Kaufman L (1984) Banded eigenvalue solvers on vector machines *ACM Trans. Math. Software* **10** 73–86

Parlett B N (1998) *The Symmetric Eigenvalue Problem* SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **vect** – CHARACTER(1)

Indicates whether Q is to be returned.

vect = 'V'
: Q is returned.

vect = 'U'
: Q is updated (and the array **q** must contain a matrix on entry).

vect = 'N'
: Q is not required.

Constraint: **vect** = 'V', 'U' or 'N'.

2: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'
: The upper triangular part of A is stored.

uplo = 'L'
The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **kd** – INTEGER

If **uplo** = 'U', the number of superdiagonals, k_d, of the matrix A.

If **uplo** = 'L', the number of subdiagonals, k_d, of the matrix A.

Constraint: **kd** ≥ 0.

4: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least $\max(1, \mathbf{kd}+1)$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The upper or lower triangle of the n by n symmetric band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

5: **q**$(ldq,:)$ – REAL (KIND=nag_wp) array

The first dimension, ldq, of the array **q** must satisfy

if **vect** = 'V' or 'U', $ldq \geq \max(1, \mathbf{n})$;
if **vect** = 'N', $ldq \geq 1$.

The second dimension of the array **q** must be at least $\max(1, \mathbf{n})$ if **vect** = 'V' or 'U' and at least 1 if **vect** = 'N'.

If **vect** = 'U', **q** must contain the matrix formed in a previous stage of the reduction (for example, the reduction of a banded symmetric-definite generalized eigenproblem); otherwise **q** need not be set.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **ab** and the second dimension of the array **ab**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** will be $\max(1, \mathbf{kd}+1)$.

The second dimension of the array **ab** will be $\max(1, \mathbf{n})$.

ab stores values generated during the reduction to tridiagonal form.

The first superdiagonal or subdiagonal and the diagonal of the tridiagonal matrix T are returned in **ab** using the same storage format as described above.

2: **d**(**n**) – REAL (KIND=nag_wp) array

The diagonal elements of the tridiagonal matrix T.

3: **e**(**n** − **1**) – REAL (KIND=nag_wp) array

The off-diagonal elements of the tridiagonal matrix T.

4: **q**(ldq, :) – REAL (KIND=nag_wp) array

The first dimension, ldq, of the array **q** will be

if **vect** = 'V' or 'U', $ldq = \max(1, \mathbf{n})$;
if **vect** = 'N', $ldq = 1$.

The second dimension of the array **q** will be $\max(1, \mathbf{n})$ if **vect** = 'V' or 'U' and at least 1 if **vect** = 'N'.

If **vect** = 'V' or 'U', the n by n matrix Q.

If **vect** = 'N', **q** is not referenced.

5: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **vect**, 2: **uplo**, 3: **n**, 4: **kd**, 5: **ab**, 6: **ldab**, 7: **d**, 8: **e**, 9: **q**, 10: **ldq**, 11: **work**, 12: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A + E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $6n^2k$ if **vect** = 'N' with $3n^3(k-1)/k$ additional operations if **vect** = 'V'.

The complex analogue of this function is nag_lapack_zhbtrd (f08hs).

9 Example

This example computes all the eigenvalues and eigenvectors of the matrix A, where

$$A = \begin{pmatrix} 4.99 & 0.04 & 0.22 & 0.00 \\ 0.04 & 1.05 & -0.79 & 1.04 \\ 0.22 & -0.79 & -2.31 & -1.30 \\ 0.00 & 1.04 & -1.30 & -0.43 \end{pmatrix}.$$

Here A is symmetric and is treated as a band matrix. The program first calls nag_lapack_dsbtrd (f08he) to reduce A to tridiagonal form T, and to form the orthogonal matrix Q; the results are then passed to nag_lapack_dsteqr (f08je) which computes the eigenvalues and eigenvectors of A.

9.1 Program Text

```
function f08he_example

fprintf('f08he example results\n\n');

% Symmetric band matrix A, stored on symmetric banded format
uplo = 'L';
kd   = nag_int(2);
n    = nag_int(4);
ab   = [4.99,  1.05, -2.31, -0.43;
        0.04, -0.79, -1.30,  0;
        0.22,  1.04,  0,     0];

% Reduce A to tridiagonal form and compute Q
vect = 'V';
q = zeros(n, n);
[apf, d, e, q, info] = f08he( ...
                              vect, uplo, kd, ab, q);

% Calculate eigenvalues and eigenvectors of A
[w, ~, z, info] = f08je( ...
                         vect, d, e, 'z', q);

% Normalize eigenvectors: largest element positive
for j = 1:n
  [~,k] = max(abs(z(:,j)));
  if z(k,j) < 0;
    z(:,j) = -z(:,j);
  end
end

disp('Eigenvalues');
disp(w');
disp('Eigenvectors');
disp(z);
```

9.2 Program Results

```
f08he example results

Eigenvalues
   -2.9943   -0.7000    1.9974    4.9969

Eigenvectors
   -0.0251    0.0162    0.0113    0.9995
    0.0656   -0.5859    0.8077    0.0020
    0.9002   -0.3135   -0.3006    0.0311
    0.4298    0.7471    0.5070   -0.0071
```