NAG Toolbox

nag_lapack_dopgtr (f08gf)

1 Purpose

nag_lapack_dopgtr (f08gf) generates the real orthogonal matrix Q, which was determined by nag_lapack_dsptrd (f08ge) when reducing a symmetric matrix to tridiagonal form.

2 Syntax

```
[q, info] = nag_lapack_dopgtr(uplo, n, ap, tau)
[q, info] = f08gf(uplo, n, ap, tau)
```

3 Description

nag_lapack_dopgtr (f08gf) is intended to be used after a call to nag_lapack_dsptrd (f08ge), which reduces a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = QTQ^{\mathrm{T}}$. nag_lapack_dsptrd (f08ge) represents the orthogonal matrix Q as a product of $n-1$ elementary reflectors.

This function may be used to generate Q explicitly as a square matrix.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

This **must** be the same argument **uplo** as supplied to nag_lapack_dsptrd (f08ge).

Constraint: **uplo** = 'U' or 'L'.

2: **n** – INTEGER

n, the order of the matrix Q.

Constraint: **n** ≥ 0.

3: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

Details of the vectors which define the elementary reflectors, as returned by nag_lapack_dsptrd (f08ge).

4: **tau**(:) – REAL (KIND=nag_wp) array

The dimension of the array **tau** must be at least $\max(1, \mathbf{n}-1)$

Further details of the elementary reflectors, as returned by nag_lapack_dsptrd (f08ge).

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **q**$(ldq,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **q** will be $\max(1,\mathbf{n})$.

The second dimension of the array **q** will be $\max(1,\mathbf{n})$.

The n by n orthogonal matrix Q.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **tau**, 5: **q**, 6: **ldq**, 7: **work**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}n^3$.

The complex analogue of this function is nag_lapack_zupgtr (f08gt).

9 Example

This example computes all the eigenvalues and eigenvectors of the matrix A, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix},$$

using packed storage. Here A is symmetric and must first be reduced to tridiagonal form by nag_lapack_dsptrd (f08ge). The program then calls nag_lapack_dopgtr (f08gf) to form Q, and passes this matrix to nag_lapack_dsteqr (f08je) which computes the eigenvalues and eigenvectors of A.

9.1 Program Text

```
     function f08gf_example

fprintf('f08gf example results\n\n');

% Symmetric matrix A stored in symmetric packed format (Lower)
uplo = 'L';
n = nag_int(4);
ap = [2.07;     3.87;     4.2;     -1.15;
              -0.21;     1.87;     0.63;
                         1.15;     2.06;
                                  -1.81];
% Reduce A to tridiagonal form
[apf, d, e, tau, info] = f08ge( ...
                                uplo, n, ap);

% Form Q
[q, info] = f08gf( ...
                  uplo, n, apf, tau);

% Calculate eigenvalues and eigenvectors
compz = 'Vectors';
[w, ~, z, info] = f08je( ...
                         compz, d, e, 'z', q);

disp('Eigenvalues');
disp(w');

% Normalize eigenvectors: largest element positive
for j = 1:n
  [~,k] = max(abs(z(:,j)));
  if z(k,j) < 0;
    z(:,j) = -z(:,j);
  end
end

disp('Eigenvectors');
disp(z);
```

9.2 Program Results

```
     f08gf example results

Eigenvalues
   -5.0034   -1.9987    0.2013    8.0008

Eigenvectors
    0.5658   -0.2328   -0.3965    0.6845
   -0.3478    0.7994   -0.1780    0.4564
   -0.4740   -0.4087    0.5381    0.5645
    0.5781    0.3737    0.7221    0.0676
```