NAG Toolbox

nag_lapack_dspevd (f08gc)

1 Purpose

nag_lapack_dspevd (f08gc) computes all the eigenvalues and, optionally, all the eigenvectors of a real symmetric matrix held in packed storage. If the eigenvectors are requested, then it uses a divide-and-conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal–Walker–Kahan variant of the QL or QR algorithm.

2 Syntax

```
[ap, w, z, info] = nag_lapack_dspevd(job, uplo, n, ap)
[ap, w, z, info] = f08gc(job, uplo, n, ap)
```

3 Description

nag_lapack_dspevd (f08gc) computes all the eigenvalues and, optionally, all the eigenvectors of a real symmetric matrix A (held in packed storage). In other words, it can compute the spectral factorization of A as

$$A = Z \Lambda Z^{\mathrm{T}},$$

where Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the orthogonal matrix whose columns are the eigenvectors z_i. Thus

$$A z_i = \lambda_i z_i, \quad i = 1, 2, \ldots, n.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – CHARACTER(1)

Indicates whether eigenvectors are computed.

job = 'N'
Only eigenvalues are computed.

job = 'V'
Eigenvalues and eigenvectors are computed.

Constraint: **job** = 'N' or 'V'.

2: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'
The upper triangular part of A is stored.

uplo = 'L'
The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **n** – INTEGER

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$

The upper or lower triangle of the n by n symmetric matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + j(j-1)/2)$ for $i \leq j$;

if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + (2n - j)(j-1)/2)$ for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** will be $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$

ap stores the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A.

2: **w**(:) – REAL (KIND=nag_wp) array

The dimension of the array **w** will be $\max(1, \mathbf{n})$

The eigenvalues of the matrix A in ascending order.

3: **z**(ldz, :) – REAL (KIND=nag_wp) array

The first dimension, ldz, of the array **z** will be

if **job** = 'V', $ldz = \max(1, \mathbf{n})$;
if **job** = 'N', $ldz = 1$.

The second dimension of the array **z** will be $\max(1, \mathbf{n})$ if **job** = 'V' and at least 1 if **job** = 'N'.

If **job** = 'V', **z** stores the orthogonal matrix Z which contains the eigenvectors of A.

If **job** = 'N', **z** is not referenced.

4: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **uplo**, 3: **n**, 4: **ap**, 5: **w**, 6: **z**, 7: **ldz**, 8: **work**, 9: **lwork**, 10: **iwork**, 11: **liwork**, 12: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

if **info** $= i$ and **job** = 'N', the algorithm failed to converge; i elements of an intermediate tridiagonal form did not converge to zero; if **info** $= i$ and **job** = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and column $i/(\mathbf{n}+1)$ through $i \bmod (\mathbf{n}+1)$.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A+E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Further Comments

The complex analogue of this function is nag_lapack_zhpevd (f08gq).

9 Example

This example computes all the eigenvalues and eigenvectors of the symmetric matrix A, where

$$A = \begin{pmatrix} 1.0 & 2.0 & 3.0 & 4.0 \\ 2.0 & 2.0 & 3.0 & 4.0 \\ 3.0 & 3.0 & 3.0 & 4.0 \\ 4.0 & 4.0 & 4.0 & 4.0 \end{pmatrix}.$$

9.1 Program Text

```
     function f08gc_example

fprintf('f08gc example results\n\n');

% A is symmetric matrix stored in symmetric (Lower) packed format
uplo = 'L';
n = nag_int(4);
ap = [1;     2;     3;     4;
             2;     3;     4;
                    3;     4;
                           4];

% Eigenvalues and vectors of A
job = 'Vectors';
[apf, w, z, info] = f08gc( ...
                           job, uplo, n, ap);

disp('Eigenvalues');
disp(w');

% Normalize eigenvectors: largest element positive
for j = 1:n
  [~,k] = max(abs(z(:,j)));
```

```
  if z(k,j) < 0;
    z(:,j) = -z(:,j);
  end
end

disp('Eigenvectors');
disp(z);
```

9.2 Program Results

```
     f08gc example results

Eigenvalues
   -2.0531   -0.5146   -0.2943   12.8621

Eigenvectors
    0.7003   -0.5144   -0.2767    0.4103
    0.3592    0.4851    0.6634    0.4422
   -0.1569    0.5420   -0.6504    0.5085
   -0.5965   -0.4543    0.2457    0.6144
```