NAG Toolbox

nag_lapack_dspev (f08ga)

1 Purpose

nag_lapack_dspev (f08ga) computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric matrix A in packed storage.

2 Syntax

```
[ap, w, z, info] = nag_lapack_dspev(jobz, uplo, n, ap)
[ap, w, z, info] = f08ga(jobz, uplo, n, ap)
```

3 Description

The symmetric matrix A is first reduced to tridiagonal form, using orthogonal similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – CHARACTER(1)

Indicates whether eigenvectors are computed.

jobz = 'N'
Only eigenvalues are computed.

jobz = 'V'
Eigenvalues and eigenvectors are computed.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo** – CHARACTER(1)

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **n** – INTEGER

n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

The upper or lower triangle of the n by n symmetric matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + j(j-1)/2)$ for $i \leq j$;

if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + (2n-j)(j-1)/2)$ for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** will be $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

ap stores the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A.

2: **w**(**n**) – REAL (KIND=nag_wp) array

The eigenvalues in ascending order.

3: **z**(ldz, :) – REAL (KIND=nag_wp) array

The first dimension, ldz, of the array **z** will be

if **jobz** = 'V', $ldz = \max(1, \mathbf{n})$;
otherwise $ldz = 1$.

The second dimension of the array **z** will be $\max(1, \mathbf{n})$ if **jobz** = 'V' and 1 otherwise.

If **jobz** = 'V', **z** contains the orthonormal eigenvectors of the matrix A, with the ith column of Z holding the eigenvector associated with $\mathbf{w}(i)$.

If **jobz** = 'N', **z** is not referenced.

4: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **ap**, 5: **w**, 6: **z**, 7: **ldz**, 8: **work**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3.

The complex analogue of this function is nag_lapack_zhpev (f08gn).

9 Example

This example finds all the eigenvalues of the symmetric matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix},$$

together with approximate error bounds for the computed eigenvalues.

9.1 Program Text

```
     function f08ga_example

fprintf('f08ga example results\n\n');

% A is symmetric matrix stored in symmetric (Upper) packed format
uplo = 'U';
n = nag_int(4);
ap = [1;
      2;   2;
      3;   3;   3;
      4;   4;   4;   4];

% Eigenvalues of A only
jobz = 'No vectors';
[apf, w, ~, info] = f08ga( ...
                           jobz, uplo, n, ap);

disp('Eigenvalues');
disp(w');

% Eigenvalue error bound
errbnd = x02aj*max(abs(w(1)),abs(w(end)));
disp('Error estimate for the eigenvalues');
fprintf('%11.1e\n',errbnd);
```

9.2 Program Results

```
     f08ga example results

Eigenvalues
   -2.0531   -0.5146   -0.2943   12.8621

Error estimate for the eigenvalues
    1.4e-15
```