NAG Toolbox

nag_lapack_zhetrd (f08fs)

1 Purpose

nag_lapack_zhetrd (f08fs) reduces a complex Hermitian matrix to tridiagonal form.

2 Syntax

```
[a, d, e, tau, info] = nag_lapack_zhetrd(uplo, a, 'n', n)
[a, d, e, tau, info] = f08fs(uplo, a, 'n', n)
```

3 Description

nag_lapack_zhetrd (f08fs) reduces a complex Hermitian matrix A to real symmetric tridiagonal form T by a unitary similarity transformation: $A = QTQ^{\mathrm{H}}$.

The matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 9).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'
The upper triangular part of A is stored.

uplo = 'L'
The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n Hermitian matrix A.

If **uplo** = 'U', the upper triangular part of a must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of a must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: $\mathbf{a}(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{n})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

a stores the tridiagonal matrix T and details of the unitary matrix Q as specified by **uplo**.

2: $\mathbf{d}(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **d** will be $\max(1,\mathbf{n})$

The diagonal elements of the tridiagonal matrix T.

3: $\mathbf{e}(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **e** will be $\max(1,\mathbf{n}-1)$

The off-diagonal elements of the tridiagonal matrix T.

4: $\mathbf{tau}(:)$ – COMPLEX (KIND=nag_wp) array

The dimension of the array **tau** will be $\max(1,\mathbf{n}-1)$

Further details of the unitary matrix Q.

5: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **d**, 6: **e**, 7: **tau**, 8: **work**, 9: **lwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A+E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{16}{3}n^3$.

To form the unitary matrix Q nag_lapack_zhetrd (f08fs) may be followed by a call to nag_lapack_zungtr (f08ft):

```
[a, info] = f08ft(uplo, a, tau);
```

To apply Q to an n by p complex matrix C nag_lapack_zhetrd (f08fs) may be followed by a call to nag_lapack_zunmtr (f08fu). For example,

```
[c, info] = f08fu('Left', uplo, 'No Transpose', a, tau, c);
```

forms the matrix product QC.

The real analogue of this function is nag_lapack_dsytrd (f08fe).

9 Example

This example reduces the matrix A to tridiagonal form, where

$$A = \begin{pmatrix} -2.28 + 0.00i & 1.78 - 2.03i & 2.26 + 0.10i & -0.12 + 2.53i \\ 1.78 + 2.03i & -1.12 + 0.00i & 0.01 + 0.43i & -1.07 + 0.86i \\ 2.26 - 0.10i & 0.01 - 0.43i & -0.37 + 0.00i & 2.31 - 0.92i \\ -0.12 - 2.53i & -1.07 - 0.86i & 2.31 + 0.92i & -0.73 + 0.00i \end{pmatrix}.$$

9.1 Program Text

```
function f08fs_example

fprintf('f08fs example results\n\n');

% A --> QTQ^H, for tridiagonal T
uplo = 'L';
a = [-2.28 + 0.00i,  0.00 + 0i,     0    + 0i,     0    + 0i;
      1.78 + 2.03i, -1.12 + 0i,     0    + 0i,     0    + 0i;
      2.26 - 0.10i,  0.01 - 0.43i, -0.37 + 0i,     0    + 0i;
     -0.12 - 2.53i, -1.07 - 0.86i,  2.31 + 0.92i, -0.73 + 0i];

[QT, d, e, tau, info] = f08fs(uplo, a);

disp('Main diagonal of T:');
disp (d');
disp('Off diagonal of T (absolute values):');
disp (abs(e)');
```

9.2 Program Results

```
f08fs example results

Main diagonal of T:
   -2.2800   -0.1285   -0.1666   -1.9249

Off diagonal of T (absolute values):
    4.3385    2.0226    1.8023
```