NAG Toolbox

nag_lapack_dsytrd (f08fe)

1 Purpose

nag_lapack_dsytrd (f08fe) reduces a real symmetric matrix to tridiagonal form.

2 Syntax

```
[a, d, e, tau, info] = nag_lapack_dsytrd(uplo, a, 'n', n)
[a, d, e, tau, info] = f08fe(uplo, a, 'n', n)
```

3 Description

nag_lapack_dsytrd (f08fe) reduces a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = QTQ^{\mathrm{T}}$.

The matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 9).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'
The upper triangular part of A is stored.

uplo = 'L'
The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The n by n symmetric matrix A.

If **uplo** = 'U', the upper triangular part of a must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of a must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{n})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

a stores the tridiagonal matrix T and details of the orthogonal matrix Q as specified by **uplo**.

2: **d**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **d** will be $\max(1,\mathbf{n})$

The diagonal elements of the tridiagonal matrix T.

3: **e**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **e** will be $\max(1,\mathbf{n}-1)$

The off-diagonal elements of the tridiagonal matrix T.

4: **tau**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **tau** will be $\max(1,\mathbf{n}-1)$

Further details of the orthogonal matrix Q.

5: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **d**, 6: **e**, 7: **tau**, 8: **work**, 9: **lwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A+E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

8 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}n^3$.

To form the orthogonal matrix Q nag_lapack_dsytrd (f08fe) may be followed by a call to nag_lapack_dorgtr (f08ff):

```
[a, info] = f08ff(uplo, a, tau);
```

To apply Q to an n by p real matrix C nag_lapack_dsytrd (f08fe) may be followed by a call to nag_lapack_dormtr (f08fg). For example,

```
[c, info] = f08fg('Left', uplo, 'No Transpose', a, tau, c);
```

forms the matrix product QC.

The complex analogue of this function is nag_lapack_zhetrd (f08fs).

9 Example

This example reduces the matrix A to tridiagonal form, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

9.1 Program Text

```
function f08fe_example

fprintf('f08fe example results\n\n');

uplo = 'L';
a = [ 2.07,  0,    0,     0;
      3.87, -0.21, 0,     0;
      4.2,   1.87, 1.15,  0;
     -1.15,  0.63, 2.06, -1.81];
n = size(a,1);

[~, d, e, tau, info] = f08fe( ...
  uplo, a);

fprintf('Diagonal and off-diagonal elements of tridiagonal form\n\n');
fprintf('    i         D           E\n');

for j = 1:n-1
  fprintf('%5d%12.5f%12.5f\n', j, d(j), e(j));
end
fprintf('%5d%12.5f\n', n, d(n));
```

9.2 Program Results

```
f08fe example results

Diagonal and off-diagonal elements of tridiagonal form

    i         D           E
    1     2.07000    -5.82575
    2     1.47409     2.62405
    3    -0.64916     0.91627
    4    -1.69493
```