NAG Toolbox

nag_lapack_dsyevd (f08fc)

1 Purpose

nag_lapack_dsyevd (f08fc) computes all the eigenvalues and, optionally, all the eigenvectors of a real symmetric matrix. If the eigenvectors are requested, then it uses a divide-and-conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal–Walker–Kahan variant of the QL or QR algorithm.

2 Syntax

```
[a, w, info] = nag_lapack_dsyevd(job, uplo, a, 'n', n)
[a, w, info] = f08fc(job, uplo, a, 'n', n)
```

3 Description

nag_lapack_dsyevd (f08fc) computes all the eigenvalues and, optionally, all the eigenvectors of a real symmetric matrix A. In other words, it can compute the spectral factorization of A as

$$A = Z\Lambda Z^{\mathrm{T}},$$

where Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the orthogonal matrix whose columns are the eigenvectors z_i. Thus

$$Az_i = \lambda_i z_i, \quad i = 1, 2, \dots, n.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – CHARACTER(1)

Indicates whether eigenvectors are computed.

job = 'N'
Only eigenvalues are computed.

job = 'V'
Eigenvalues and eigenvectors are computed.

Constraint: **job** = 'N' or 'V'.

2: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'
The upper triangular part of A is stored.

uplo = 'L'
The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n symmetric matrix A.

If **uplo** = 'U', the upper triangular part of a must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of a must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{n})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

If **job** = 'V', **a** stores the orthogonal matrix Z which contains the eigenvectors of A.

2: **w**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **w** will be $\max(1,\mathbf{n})$

The eigenvalues of the matrix A in ascending order.

3: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **uplo**, 3: **n**, 4: **a**, 5: **lda**, 6: **w**, 7: **work**, 8: **lwork**, 9: **iwork**, 10: **liwork**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

if **info** $= i$ and **job** = 'N', the algorithm failed to converge; i elements of an intermediate tridiagonal form did not converge to zero; if **info** $= i$ and **job** = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and column $i/(\mathbf{n}+1)$ through i mod $(\mathbf{n}+1)$.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A+E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Further Comments

The complex analogue of this function is nag_lapack_zheevd (f08fq).

9 Example

This example computes all the eigenvalues and eigenvectors of the symmetric matrix A, where

$$A = \begin{pmatrix} 1.0 & 2.0 & 3.0 & 4.0 \\ 2.0 & 2.0 & 3.0 & 4.0 \\ 3.0 & 3.0 & 3.0 & 4.0 \\ 4.0 & 4.0 & 4.0 & 4.0 \end{pmatrix}.$$

9.1 Program Text

```
     function f08fc_example

fprintf('f08fc example results\n\n');

job = 'V';
uplo = 'L';
a = [1, 0, 0, 0;
     2, 2, 0, 0;
     3, 3, 3, 0;
     4, 4, 4, 4];

[z, w, info] = f08fc( ...
       job, uplo, a);

disp('Eigenvalues');
disp(w');

[ifail] = x04ca( ...
  'General', ' ', z, 'Eigenvectors');
```

9.2 Program Results

```
     f08fc example results

Eigenvalues
   -2.0531   -0.5146   -0.2943   12.8621

 Eigenvectors
          1       2       3       4
 1  -0.7003 -0.5144 -0.2767 -0.4103
 2  -0.3592  0.4851  0.6634 -0.4422
 3   0.1569  0.5420 -0.6504 -0.5085
 4   0.5965 -0.4543  0.2457 -0.6144
```