NAG Toolbox

nag_lapack_dsyev (f08fa)

1 Purpose

nag_lapack_dsyev (f08fa) computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric matrix A.

2 Syntax

```
[a, w, info] = nag_lapack_dsyev(jobz, uplo, a, 'n', n)
[a, w, info] = f08fa(jobz, uplo, a, 'n', n)
```

3 Description

The symmetric matrix A is first reduced to tridiagonal form, using orthogonal similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – CHARACTER(1)

Indicates whether eigenvectors are computed.

jobz = 'N'
Only eigenvalues are computed.

jobz = 'V'
Eigenvalues and eigenvectors are computed.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo** – CHARACTER(1)

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n symmetric matrix A.

If **uplo** $=$ 'U', the upper triangular part of a must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** $=$ 'L', the lower triangular part of a must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: $\mathbf{a}(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{n})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

If **jobz** $=$ 'V', then **a** contains the orthonormal eigenvectors of the matrix A.

If **jobz** $=$ 'N', then on exit the lower triangle (if **uplo** $=$ 'L') or the upper triangle (if **uplo** $=$ 'U') of **a**, including the diagonal, is overwritten.

2: $\mathbf{w}(\mathbf{n})$ – REAL (KIND=nag_wp) array

The eigenvalues in ascending order.

3: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **a**, 5: **lda**, 6: **w**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A+E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3.

The complex analogue of this function is nag_lapack_zheev (f08fn).

9 Example

This example finds all the eigenvalues and eigenvectors of the symmetric matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix},$$

together with approximate error bounds for the computed eigenvalues and eigenvectors.

9.1 Program Text

```
     function f08fa_example

fprintf('f08fa example results\n\n');

a = [1, 2, 3, 4;
     0, 2, 3, 4;
     0, 0, 3, 4;
     0, 0, 0, 4];
n = nag_int(size(a,1));

% Eigenvalues and eogenvectors of upper triangular A
jobz = 'Vectors';
uplo = 'Upper';
[v, w, info] = f08fa( ...
       jobz, uplo, a);

disp('Eigenvectors');
disp(v);

% Eigenvalue error bound
errbnd = x02aj*max(abs(w(1)),abs(w(end)));
% Eigenvector condition numbers
[rcondz, info] = f08fl( ...
         'Eigenvectors', n, n, w);

% Eigenvector error bounds
zerrbd = errbnd./rcondz;

disp('Error estimate for the eigenvalues');
fprintf('%12.1e\n',errbnd);
disp('Error estimates for the eigenvectors');
fprintf('%12.1e',zerrbd);
fprintf('\n');
```

9.2 Program Results

```
     f08fa example results

Eigenvectors
    0.7003   -0.5144    0.2767    0.4103
    0.3592    0.4851   -0.6634    0.4422
   -0.1569    0.5420    0.6504    0.5085
   -0.5965   -0.4543   -0.2457    0.6144
```

```
Error estimate for the eigenvalues
     1.4e-15
Error estimates for the eigenvectors
     9.3e-16     6.5e-15     6.5e-15     1.1e-16
```
