NAG Toolbox

nag_lapack_zunglq (f08aw)

1 Purpose

nag_lapack_zunglq (f08aw) generates all or part of the complex unitary matrix Q from an LQ factorization computed by nag_lapack_zgelqf (f08av).

2 Syntax

```
[a, info] = nag_lapack_zunglq(a, tau, 'm', m, 'n', n, 'k', k)
[a, info] = f08aw(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

nag_lapack_zunglq (f08aw) is intended to be used after a call to nag_lapack_zgelqf (f08av), which performs an LQ factorization of a complex matrix A. The unitary matrix Q is represented as a product of elementary reflectors.

This function may be used to generate Q explicitly as a square matrix, or to form only its leading rows.

Usually Q is determined from the LQ factorization of a p by n matrix A with $p \leq n$. The whole of Q may be computed by:

```
[a, info] = f08aw(a, tau);
```

(note that the array **a** must have at least n rows) or its leading p rows by:

```
[a, info] = f08aw(a(1:p,:), tau);
```

The rows of Q returned by the last call form an orthonormal basis for the space spanned by the rows of A; thus nag_lapack_zgelqf (f08av) followed by nag_lapack_zunglq (f08aw) can be used to orthogonalize the rows of A.

The information returned by the LQ factorization functions also yields the LQ factorization of the leading k rows of A, where $k < p$. The unitary matrix arising from this factorization can be computed by:

```
[a, info] = f08aw(a, tau);
```

or its leading k rows by:

```
[a, info] = f08aw(a(1:k,:), tau);
```

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

Details of the vectors which define the elementary reflectors, as returned by nag_lapack_zgelqf (f08av).

2: **tau**(:) – COMPLEX (KIND=nag_wp) array

The dimension of the array **tau** must be at least $\max(1, \mathbf{k})$

Further details of the elementary reflectors, as returned by nag_lapack_zgelqf (f08av).

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix Q.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix Q.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

3: **k** – INTEGER

Default: the dimension of the array **tau**.

k, the number of elementary reflectors whose product defines the matrix Q.

Constraint: $\mathbf{m} \geq \mathbf{k} \geq 0$.

5.3 Output Parameters

1: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The m by n matrix Q.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly unitary matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $16mnk - 8(m+n)k^2 + \frac{16}{3}k^3$; when $m = k$, the number is approximately $\frac{8}{3}m^2(3n - m)$.

The real analogue of this function is nag_lapack_dorglq (f08aj).

9 Example

This example forms the leading 4 rows of the unitary matrix Q from the LQ factorization of the matrix A, where

$$A = \begin{pmatrix} 0.28 - 0.36i & 0.50 - 0.86i & -0.77 - 0.48i & 1.58 + 0.66i \\ -0.50 - 1.10i & -1.21 + 0.76i & -0.32 - 0.24i & -0.27 - 1.15i \\ 0.36 - 0.51i & -0.07 + 1.33i & -0.75 + 0.47i & -0.08 + 1.01i \end{pmatrix}.$$

The rows of Q form an orthonormal basis for the space spanned by the rows of A.

9.1 Program Text

```
     function f08aw_example

fprintf('f08aw example results\n\n');

a = [ 0.28 - 0.36i,  0.50 - 0.86i, -0.77 - 0.48i,  1.58 + 0.66i;
     -0.50 - 1.10i, -1.21 + 0.76i, -0.32 - 0.24i, -0.27 - 1.15i;
      0.36 - 0.51i, -0.07 + 1.33i, -0.75 + 0.47i, -0.08 + 1.01i];

% Compute the LQ factorization of A
[lq, tau, info] = f08av(a);

% Form Q from LQ
[q, info] = f08aw(lq, tau);

disp('Unitary Matrix Q:');
disp(q);
```

9.2 Program Results

```
     f08aw example results

Unitary Matrix Q:
  -0.1258 + 0.1618i  -0.2247 + 0.3864i   0.3460 + 0.2157i  -0.7099 - 0.2966i
  -0.1163 - 0.6380i  -0.3240 + 0.4272i  -0.1995 - 0.5009i  -0.0323 - 0.0162i
  -0.4607 + 0.1090i   0.2171 - 0.4062i   0.2733 - 0.6106i  -0.0994 - 0.3261i
```