NAG Toolbox

nag_lapack_dorgqr (f08af)

1 Purpose

nag_lapack_dorgqr (f08af) generates all or part of the real orthogonal matrix Q from a QR factorization computed by nag_lapack_dgeqrf (f08ae), nag_lapack_dgeqpf (f08be) or nag_lapack_dgeqp3 (f08bf).

2 Syntax

```
[a, info] = nag_lapack_dorgqr(a, tau, 'm', m, 'n', n, 'k', k)
[a, info] = f08af(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

nag_lapack_dorgqr (f08af) is intended to be used after a call to nag_lapack_dgeqrf (f08ae), nag_lapack_dgeqpf (f08be) or nag_lapack_dgeqp3 (f08bf). which perform a QR factorization of a real matrix A. The orthogonal matrix Q is represented as a product of elementary reflectors.

This function may be used to generate Q explicitly as a square matrix, or to form only its leading columns.

Usually Q is determined from the QR factorization of an m by p matrix A with $m \geq p$. The whole of Q may be computed by:

```
[a, info] = f08af(a, tau, 'k', p);
```

(note that the array **a** must have m columns) or its leading p columns by:

```
[a, info] = f08af(a(:,1:p), tau, 'k', p);
```

The columns of Q returned by the last call form an orthonormal basis for the space spanned by the columns of A; thus nag_lapack_dgeqrf (f08ae) followed by nag_lapack_dorgqr (f08af) can be used to orthogonalize the columns of A.

The information returned by the QR factorization functions also yields the QR factorization of the leading k columns of A, where $k < p$. The orthogonal matrix arising from this factorization can be computed by:

```
[a, info] = f08af(a, tau, 'k', k);
```

or its leading k columns by:

```
[a, info] = f08af(a(:,1:p), tau, 'k', k);
```

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: $\mathbf{a}(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

Details of the vectors which define the elementary reflectors, as returned by nag_lapack_dgeqrf (f08ae), nag_lapack_dgeqpf (f08be) or nag_lapack_dgeqp3 (f08bf).

2: **tau**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **tau** must be at least $\max(1, \mathbf{k})$

Further details of the elementary reflectors, as returned by nag_lapack_dgeqrf (f08ae), nag_lapack_dgeqpf (f08be) or nag_lapack_dgeqp3 (f08bf).

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the order of the orthogonal matrix Q.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix Q.

Constraint: $\mathbf{m} \geq \mathbf{n} \geq 0$.

3: **k** – INTEGER

Default: the dimension of the array **tau**.

k, the number of elementary reflectors whose product defines the matrix Q.

Constraint: $\mathbf{n} \geq \mathbf{k} \geq 0$.

5.3 Output Parameters

1: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The m by n matrix Q.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $4mnk - 2(m+n)k^2 + \frac{4}{3}k^3$; when $n = k$, the number is approximately $\frac{2}{3}n^2(3m - n)$.

The complex analogue of this function is nag_lapack_zungqr (f08at).

9 Example

This example forms the leading 4 columns of the orthogonal matrix Q from the QR factorization of the matrix A, where

$$A = \begin{pmatrix} -0.57 & -1.28 & -0.39 & 0.25 \\ -1.93 & 1.08 & -0.31 & -2.14 \\ 2.30 & 0.24 & 0.40 & -0.35 \\ -1.93 & 0.64 & -0.66 & 0.08 \\ 0.15 & 0.30 & 0.15 & -2.13 \\ -0.02 & 1.03 & -1.43 & 0.50 \end{pmatrix}.$$

The columns of Q form an orthonormal basis for the space spanned by the columns of A.

9.1 Program Text

```
     function f08af_example

fprintf('f08af example results\n\n');

a = [-0.57, -1.28, -0.39,  0.25;
     -1.93,  1.08, -0.31, -2.14;
      2.30,  0.24,  0.40, -0.35;
     -1.93,  0.64, -0.66,  0.08;
      0.15,  0.30,  0.15, -2.13;
     -0.02,  1.03, -1.43,  0.50];

% Compute the QR Factorisation of A
[a, tau, info] = f08ae(a);

% Generate q
[q, info] = f08af(a, tau);

disp('Orthogonal factor Q');
disp(q);
```

9.2 Program Results

```
     f08af example results

Orthogonal factor Q
   -0.1576    0.6744   -0.4571    0.4489
   -0.5335   -0.3861    0.2583    0.3898
    0.6358   -0.2928    0.0165    0.1930
   -0.5335   -0.1692   -0.0834   -0.2350
    0.0415   -0.1593    0.1475    0.7436
   -0.0055   -0.5064   -0.8339    0.0335
```