NAG Toolbox

nag_lapack_zpftrf (f07wr)

1 Purpose

nag_lapack_zpftrf (f07wr) computes the Cholesky factorization of a complex Hermitian positive definite matrix stored in Rectangular Full Packed (RFP) format.

2 Syntax

```
[ar, info] = nag_lapack_zpftrf(transr, uplo, n, ar)
[ar, info] = f07wr(transr, uplo, n, ar)
```

3 Description

nag_lapack_zpftrf (f07wr) forms the Cholesky factorization of a complex Hermitian positive definite matrix A either as $A = U^{H}U$ if **uplo** = 'U' or $A = LL^{H}$ if **uplo** = 'L', where U is an upper triangular matrix and L is a lower triangular, stored in RFP format. The RFP storage format is described in Section 3.2.3 in the F07 Chapter Introduction.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville http://www.netlib.org/lapack/lawnspdf/lawn14.pdf

Gustavson F G, Waśniewski J, Dongarra J J and Langou J (2010) Rectangular full packed format for Cholesky's algorithm: factorization, solution, and inversion *ACM Trans. Math. Software* **37, 2**

5 Parameters

5.1 Compulsory Input Parameters

1: **transr** – CHARACTER(1)

Specifies whether the normal RFP representation of A or its conjugate transpose is stored.

transr = 'N'
The matrix A is stored in normal RFP format.

transr = 'C'
The conjugate transpose of the RFP representation of the matrix A is stored.

Constraint: **transr** = 'N' or 'C'.

2: **uplo** – CHARACTER(1)

Specifies whether the upper or lower triangular part of A is stored.

uplo = 'U'
The upper triangular part of A is stored, and A is factorized as $U^{H}U$, where U is upper triangular.

uplo = 'L'
The lower triangular part of A is stored, and A is factorized as LL^{H}, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **n** – INTEGER

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: $\mathbf{ar}(\mathbf{n} \times (\mathbf{n} + \mathbf{1})/\mathbf{2})$ – COMPLEX (KIND=nag_wp) array

The upper or lower triangular part (as specified by **uplo**) of the n by n Hermitian matrix A, in either normal or transposed RFP format (as specified by **transr**). The storage format is described in detail in Section 3.2.3 in the F07 Chapter Introduction.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: $\mathbf{ar}(\mathbf{n} \times (\mathbf{n} + \mathbf{1})/\mathbf{2})$ – COMPLEX (KIND=nag_wp) array

If $\mathbf{info} = 0$, the factor U or L from the Cholesky factorization $A = U^{\mathrm{H}}U$ or $A = LL^{\mathrm{H}}$, in the same storage format as A.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

$\mathbf{info} > 0$

The leading minor of order $\langle value \rangle$ is not positive definite and the factorization could not be completed. Hence A itself is not positive definite. This may indicate an error in forming the matrix A. There is no function specifically designed to factorize a Hermitian matrix stored in RFP format which is not positive definite; the matrix must be treated as a full Hermitian matrix, by calling nag_lapack_zhetrf (f07mr).

7 Accuracy

If $\mathbf{uplo} = $ 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon|U^{\mathrm{H}}||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\mathbf{uplo} = $ 'L', a similar statement holds for the computed factor L. It follows that $|e_{ij}| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

A call to nag_lapack_zpftrf (f07wr) may be followed by calls to the functions:

nag_lapack_zpftrs (f07ws) to solve $AX = B$;

nag_lapack_zpftri (f07ww) to compute the inverse of A.

The real analogue of this function is nag_lapack_dpftrf (f07wd).

9 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

and is stored using RFP format.

9.1 Program Text

```
function f07wr_example

fprintf('f07wr example results\n\n');

% Symmetric matrix in RFP format
transr = 'n';
uplo   = 'l';
ar = [ 4.09 + 0.00i  2.33 + 0.14i;
       3.23 + 0.00i  4.29 + 0.00i;
       1.51 + 1.92i  3.58 + 0.00i;
       1.90 - 0.84i -0.23 - 1.11i;
       0.42 - 2.50i -1.18 - 1.37i];
n  = nag_int(4);
n2 = (n*(n+1))/2;
ar  = reshape(ar,[n2,1]);

% Factorize a
[ar, info] = f07wr(transr, uplo, n, ar);

if info == 0
  % Convert factor to full array form for display
  [a, info] = f01vh(transr, uplo, n, ar);
  fprintf('\n');
  ncols  = nag_int(80);
  indent = nag_int(0);
  form   = 'f7.4';
  title  = 'Factor L:';
  diag   = 'n';
  [ifail] = x04db( ...
                   uplo, diag, a, 'brackets', form, title, ...
                   'int', 'int', ncols, indent);
else
  fprintf('\na is not positive definite.\n');
end
```

9.2 Program Results

```
f07wr example results

 Factor L:
                    1                  2                  3                  4
 1  ( 1.7972, 0.0000)
 2  ( 0.8402, 1.0683) ( 1.3164, 0.0000)
 3  ( 1.0572,-0.4674) (-0.4702, 0.3131) ( 1.5604,-0.0000)
 4  ( 0.2337,-1.3910) ( 0.0834, 0.0368) ( 0.9360, 0.8105) ( 0.8713,-0.0000)
```