NAG Toolbox

nag_lapack_dpftrf (f07wd)

1 Purpose

nag_lapack_dpftrf (f07wd) computes the Cholesky factorization of a real symmetric positive definite matrix stored in Rectangular Full Packed (RFP) format.

2 Syntax

```
[ar, info] = nag_lapack_dpftrf(transr, uplo, n, ar)
[ar, info] = f07wd(transr, uplo, n, ar)
```

3 Description

nag_lapack_dpftrf (f07wd) forms the Cholesky factorization of a real symmetric positive definite matrix A either as $A = U^{\mathrm{T}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{T}}$ if **uplo** = 'L', where U is an upper triangular matrix and L is a lower triangular, stored in RFP format. The RFP storage format is described in Section 3.2.3 in the F07 Chapter Introduction.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville http://www.netlib.org/lapack/lawnspdf/lawn14.pdf

Gustavson F G, Waśniewski J, Dongarra J J and Langou J (2010) Rectangular full packed format for Cholesky's algorithm: factorization, solution, and inversion *ACM Trans. Math. Software* **37, 2**

5 Parameters

5.1 Compulsory Input Parameters

1: **transr** – CHARACTER(1)

Specifies whether the RFP representation of A is normal or transposed.

transr = 'N'
The matrix A is stored in normal RFP format.

transr = 'T'
The matrix A is stored in transposed RFP format.

Constraint: **transr** = 'N' or 'T'.

2: **uplo** – CHARACTER(1)

Specifies whether the upper or lower triangular part of A is stored.

uplo = 'U'
The upper triangular part of A is stored, and A is factorized as $U^{\mathrm{T}}U$, where U is upper triangular.

uplo = 'L'
The lower triangular part of A is stored, and A is factorized as LL^{T}, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **n** – INTEGER

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: $\mathbf{ar}(\mathbf{n} \times (\mathbf{n} + \mathbf{1})/\mathbf{2})$ – REAL (KIND=nag_wp) array

The upper or lower triangular part (as specified by **uplo**) of the n by n symmetric matrix A, in either normal or transposed RFP format (as specified by **transr**). The storage format is described in detail in Section 3.2.3 in the F07 Chapter Introduction.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: $\mathbf{ar}(\mathbf{n} \times (\mathbf{n} + \mathbf{1})/\mathbf{2})$ – REAL (KIND=nag_wp) array

If $\mathbf{info} = 0$, the factor U or L from the Cholesky factorization $A = U^{\mathrm{T}}U$ or $A = LL^{\mathrm{T}}$, in the same storage format as A.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

$\mathbf{info} > 0$ (*warning*)

The leading minor of order $\langle$*value*$\rangle$ is not positive definite and the factorization could not be completed. Hence A itself is not positive definite. This may indicate an error in forming the matrix A. There is no function specifically designed to factorize a symmetric matrix stored in RFP format which is not positive definite; the matrix must be treated as a full symmetric matrix, by calling nag_lapack_dsytrf (f07md).

7 Accuracy

If $\mathbf{uplo} =$ 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon \left|U^{\mathrm{T}}\right||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\mathbf{uplo} =$ 'L', a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to nag_lapack_dpftrf (f07wd) may be followed by calls to the functions:

nag_lapack_dpftrs (f07we) to solve $AX = B$;

nag_lapack_dpftri (f07wj) to compute the inverse of A.

The complex analogue of this function is nag_lapack_zpftrf (f07wr).

9 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix},$$

and is stored using RFP format.

9.1 Program Text

```
function f07wd_example

fprintf('f07wd example results\n\n');

% Symmetric matrix in RFP format
transr = 'n';
uplo   = 'l';
ar = [ 0.76   0.34;
       4.16   1.18;
      -3.12   5.03;
       0.56  -0.83;
      -0.10   1.18];
n  = nag_int(4);
n2 = (n*(n+1))/2;
ar  = reshape(ar,[n2,1]);

% Factorize a
[ar, info] = f07wd(transr, uplo, n, ar);

if info == 0
  % Convert factor to full array form, and print it
  [a, info] = f01vg(transr, uplo, n, ar);
  fprintf('\n');
  [ifail] = x04ca(uplo, 'n', a, 'Factor');
else
  fprintf('\na is not positive definite.\n');
end
```

9.2 Program Results

```
f07wd example results

 Factor
            1          2          3          4
 1      2.0396
 2     -1.5297     1.6401
 3      0.2746    -0.2500     0.7887
 4     -0.0490     0.6737     0.6617     0.5347
```