NAG Toolbox

nag_lapack_ztbcon (f07vu)

1 Purpose

nag_lapack_ztbcon (f07vu) estimates the condition number of a complex triangular band matrix.

2 Syntax

```
[rcond, info] = nag_lapack_ztbcon(norm_p, uplo, diag, kd, ab, 'n', n)
[rcond, info] = f07vu(norm_p, uplo, diag, kd, ab, 'n', n)
```

3 Description

nag_lapack_ztbcon (f07vu) estimates the condition number of a complex triangular band matrix A, in either the 1-norm or the ∞-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^{\mathrm{T}})$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – CHARACTER(1)

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'
$\kappa_1(A)$ is estimated.

norm_p = 'I'
$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **uplo** – CHARACTER(1)

Specifies whether A is upper or lower triangular.

uplo = 'U'
A is upper triangular.

uplo = 'L'
A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **diag** – CHARACTER(1)

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'
A is a nonunit triangular matrix.

diag = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **kd** – INTEGER

k_d, the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0.

5: **ab**$(ldab, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **kd** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The n by n triangular band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**$(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**$(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_ztbcon (f07vu) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{H}}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8nk$ real floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to nag_lapack_ztbtrs (f07vs) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is nag_lapack_dtbcon (f07vg).

9 Example

This example estimates the condition number in the 1-norm of the matrix A, where

$$A = \begin{pmatrix} -1.94 + 4.43i & 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i \\ -3.39 + 3.44i & 4.12 - 4.27i & 0.00 + 0.00i & 0.00 + 0.00i \\ 1.62 + 3.68i & -1.84 + 5.53i & 0.43 - 2.66i & 0.00 + 0.00i \\ 0.00 + 0.00i & -2.77 - 1.93i & 1.74 - 0.04i & 0.44 + 0.10i \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with two subdiagonals. The true condition number in the 1-norm is 71.51.

9.1 Program Text

```
     function f07vu_example

fprintf('f07vu example results\n\n');

% Condition number of A, where A is complex lower triangular banded
% and stored in triangular/symmetric banded format
kd = nag_int(2);
ab = [-1.94 + 4.43i,   4.12 - 4.27i,  0.43 - 2.66i,  0.44 + 0.10i;
      -3.39 + 3.44i,  -1.84 + 5.53i,  1.74 - 0.04i,  0    + 0i;
       1.62 + 3.68i,  -2.77 - 1.93i,  0    + 0i,     0    + 0i];
b = [ -8.86 -  3.88i, -24.09 -  5.27i;
     -15.57 - 23.41i, -57.97 +  8.14i;
      -7.63 + 22.78i,  19.09 - 29.51i;
     -14.74 -  2.40i,  19.17 + 21.33i];

% Reciprocal condition number
norm_p = '1';
uplo = 'L';
diag = 'N';
[rcond, info] = f07vu( ...
                       norm_p, uplo, diag, kd, ab);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

9.2 Program Results

```
     f07vu example results

Estimate of condition number =  3.35e+01
```