NAG Toolbox

nag_lapack_ztrtrs (f07ts)

1 Purpose

nag_lapack_ztrtrs (f07ts) solves a complex triangular system of linear equations with multiple right-hand sides, $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$.

2 Syntax

```
[b, info] = nag_lapack_ztrtrs(uplo, trans, diag, a, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07ts(uplo, trans, diag, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_ztrtrs (f07ts) solves a complex triangular system of linear equations $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies whether A is upper or lower triangular.

uplo = 'U'
A is upper triangular.

uplo = 'L'
A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans** – CHARACTER(1)

Indicates the form of the equations.

trans = 'N'
The equations are of the form $AX = B$.

trans = 'T'
The equations are of the form $A^{\mathrm{T}}X = B$.

trans = 'C'
The equations are of the form $A^{\mathrm{H}}X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag** – CHARACTER(1)

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'
A is a nonunit triangular matrix.

diag = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n triangular matrix A.

If **uplo** = 'U', a is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', a is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of a are assumed to be 1, and are not referenced.

5: **b**$(ldb,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1,\mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **b**$(ldb,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1,\mathbf{n})$.

The second dimension of the array **b** will be $\max(1,\mathbf{nrhs\_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element $\langle value \rangle$ of the diagonal is exactly zero. A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon, \qquad \text{provided} \qquad c(n)\,\mathrm{cond}(A, x)\epsilon < 1,$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A, x) \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}(A^{\mathrm{H}})$, which is the same as $\mathrm{cond}(A^{\mathrm{T}})$, to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_ztrrfs (f07tv), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_lapack_ztrcon (f07tu) with **norm_p** = 'I'.

8 Further Comments

The total number of real floating-point operations is approximately $4n^2r$.

The real analogue of this function is nag_lapack_dtrtrs (f07te).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 4.78 + 4.56i & 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.00 - 0.30i & -4.11 + 1.25i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.89 - 1.34i & 2.36 - 4.25i & 4.15 + 0.80i & 0.00 + 0.00i \\ -1.89 + 1.15i & 0.04 - 3.69i & -0.02 + 0.46i & 0.33 - 0.26i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -14.78 - 32.36i & -18.02 + 28.46i \\ 2.98 - 2.14i & 14.22 + 15.42i \\ -20.96 + 17.06i & 5.62 + 35.89i \\ 9.54 + 9.91i & -16.46 - 1.73i \end{pmatrix}.$$

9.1 Program Text

```
     function f07ts_example

fprintf('f07ts example results\n\n');

% Solve AX=B where A is Lower triangular
a = [  4.78 +  4.56i,   0    +  0i,     0    + 0i,    0    + 0i;
       2.00 -  0.30i,  -4.11 +  1.25i,  0    + 0i,    0    + 0i;
       2.89 -  1.34i,   2.36 -  4.25i,  4.15 + 0.8i,  0    + 0i;
      -1.89 +  1.15i,   0.04 -  3.69i, -0.02 + 0.46i, 0.33 - 0.26i];
b = [-14.78 - 32.36i, -18.02 + 28.46i;
       2.98 -  2.14i,  14.22 + 15.42i;
     -20.96 + 17.06i,   5.62 + 35.89i;
       9.54 +  9.91i, -16.46 -  1.73i];

% Solve
uplo  = 'L';
trans = 'N';
diag  = 'N';
[x, info] = f07ts( ...
                  uplo, trans, diag, a, b);

% Display solution
disp('Solution(s)');
disp(x);
```

9.2 Program Results

```
     f07ts example results

Solution(s)
  -5.0000 - 2.0000i   1.0000 + 5.0000i
  -3.0000 - 1.0000i  -2.0000 - 2.0000i
   2.0000 + 1.0000i   3.0000 + 4.0000i
   4.0000 + 3.0000i   4.0000 - 3.0000i
```