NAG Toolbox

nag_lapack_zsytrs (f07ns)

1 Purpose

nag_lapack_zsytrs (f07ns) solves a complex symmetric system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by nag_lapack_zsytrf (f07nr).

2 Syntax

```
[b, info] = nag_lapack_zsytrs(uplo, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07ns(uplo, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_zsytrs (f07ns) is used to solve a complex symmetric system of linear equations $AX = B$, this function must be preceded by a call to nag_lapack_zsytrf (f07nr) which computes the Bunch–Kaufman factorization of A.

If **uplo** $=$ 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

If **uplo** $=$ 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo $=$ 'U'
$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

uplo $=$ 'L'
$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** $=$ 'U' or 'L'.

2: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

Details of the factorization of A, as returned by nag_lapack_zsytrf (f07nr).

3: **ipiv**(:) – INTEGER array

The dimension of the array **ipiv** must be at least max$(1,$ **n**$)$

Details of the interchanges and the block structure of D, as returned by nag_lapack_zsytrf (f07nr).

4: **b**$(ldb,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** must be at least max$(1,$ **n**$)$.

The second dimension of the array **b** must be at least max$(1,$ **nrhs_p**$)$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the arrays **a**, **ipiv**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0.

5.3 Output Parameters

1: **b**$(ldb,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** will be max$(1,$ **n**$)$.

The second dimension of the array **b** will be max$(1,$ **nrhs_p**$)$.

The n by r solution matrix X.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x=b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon P|U||D||U^{\mathrm{T}}|P^{\mathrm{T}}$;

if **uplo** = 'L', $|E| \leq c(n)\epsilon P|L||D||L^{\mathrm{T}}|P^{\mathrm{T}}$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_zsyrfs (f07nv), and an estimate for $\kappa_\infty(A)$ ($=\kappa_1(A)$) can be obtained by calling nag_lapack_zsycon (f07nu).

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to nag_lapack_zsyrfs (f07nv) to refine the solution and return an error estimate.

The real analogue of this function is nag_lapack_dsytrs (f07me).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.39-0.71i & 5.14-0.64i & -7.86-2.96i & 3.80+0.92i \\ 5.14-0.64i & 8.86+1.81i & -3.52+0.58i & 5.32-1.59i \\ -7.86-2.96i & -3.52+0.58i & -2.83-0.03i & -1.54-2.86i \\ 3.80+0.92i & 5.32-1.59i & -1.54-2.86i & -0.56+0.12i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -55.64+41.22i & -19.09-35.97i \\ -48.18+66.00i & -12.08-27.02i \\ -0.49-\ \ 1.47i & 6.95+20.49i \\ -6.43+19.24i & -4.59-35.53i \end{pmatrix}.$$

Here A is symmetric and must first be factorized by nag_lapack_zsytrf (f07nr).

9.1 Program Text

```
     function f07ns_example

fprintf('f07ns example results\n\n');

% Complex symmetrix matrix A, lower triangle stored.
uplo = 'L';
a = [-0.39 - 0.71i,  0    + 0i,     0    + 0i,     0    + 0i;
      5.14 - 0.64i,  8.86 + 1.81i,  0    + 0i,     0    + 0i;
     -7.86 - 2.96i, -3.52 + 0.58i, -2.83 - 0.03i,  0    + 0i;
      3.80 + 0.92i,  5.32 - 1.59i, -1.54 - 2.86i, -0.56 + 0.12i];

%Factorize A
[af, ipiv, info] = f07nr( ...
                         uplo, a);

% RHS
b = [ -55.64 + 41.22i, -19.09 - 35.97i;
      -48.18 + 66.00i, -12.08 - 27.02i;
       -0.49 -  1.47i,   6.95 + 20.49i;
       -6.43 + 19.24i,  -4.59 - 35.53i];

% Solve Ax=b
```

```
[x, info] = f07ns( ...
                   uplo, af, ipiv, b);

disp('Solution:');
disp(x);
```

9.2 Program Results

```
     f07ns example results

Solution:
   1.0000 - 1.0000i  -2.0000 - 1.0000i
  -2.0000 + 5.0000i   1.0000 - 3.0000i
   3.0000 - 2.0000i   3.0000 + 2.0000i
  -4.0000 + 3.0000i  -1.0000 + 1.0000i
```