# NAG Toolbox

# nag_lapack_zhecon (f07mu)

## 1 Purpose

nag_lapack_zhecon (f07mu) estimates the condition number of a complex Hermitian indefinite matrix $A$, where $A$ has been factorized by nag_lapack_zhetrf (f07mr).

## 2 Syntax

```
[rcond, info] = nag_lapack_zhecon(uplo, a, ipiv, anorm, 'n', n)
[rcond, info] = f07mu(uplo, a, ipiv, anorm, 'n', n)
```

## 3 Description

nag_lapack_zhecon (f07mu) estimates the condition number (in the 1-norm) of a complex Hermitian indefinite matrix $A$:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since $A$ is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if $A$ is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a computation of $\|A\|_1$ and a call to nag_lapack_zhetrf (f07mr) to compute the Bunch–Kaufman factorization of $A$. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

## 4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how $A$ has been factorized.

**uplo** = 'U'
$A = PUDU^{\mathrm{H}}P^{\mathrm{T}}$, where $U$ is upper triangular.

**uplo** = 'L'
$A = PLDL^{\mathrm{H}}P^{\mathrm{T}}$, where $L$ is lower triangular.

*Constraint*: **uplo** = 'U' or 'L'.

2: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

Details of the factorization of $A$, as returned by nag_lapack_zhetrf (f07mr).

3: **ipiv**(:) – INTEGER array

The dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$

Details of the interchanges and the block structure of $D$, as returned by nag_lapack_zhetrf (f07mr).

4: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix $A$. **anorm** must be computed either **before** calling nag_lapack_zhetrf (f07mr) or else from a **copy** of the original matrix $A$.

*Constraint*: **anorm** $\geq 0.0$.

### 5.2 Optional Input Parameters

1: **n** – INTEGER

*Default*: the first dimension of the array **a** and the second dimension of the arrays **a**, **ipiv**.

$n$, the order of the matrix $A$.

*Constraint*: **n** $\geq 0$.

### 5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of $A$. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, $A$ is singular to working precision.

2: **info** – INTEGER

**info** $= 0$ unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

**info** $< 0$

If **info** $= -i$, argument $i$ had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The computed estimate **rcond** is never less than the true value $\rho$, and in practice is nearly always less than $10\rho$, although examples can be constructed where **rcond** is much larger.

## 8 Further Comments

A call to nag_lapack_zhecon (f07mu) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to nag_lapack_zhetrs (f07ms) with one right-hand side, because extra care is taken to avoid overflow when $A$ is approximately singular.

The real analogue of this function is nag_lapack_dsycon (f07mg).

## 9 Example

This example estimates the condition number in the 1-norm (or $\infty$-norm) of the matrix $A$, where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here $A$ is Hermitian indefinite and must first be factorized by nag_lapack_zhetrf (f07mr). The true condition number in the 1-norm is 9.10.

### 9.1 Program Text

```
     function f07mu_example

fprintf('f07mu example results\n\n');

% Hermitian indefinite matrix A (Lower triangular part stored)
uplo = 'L';
a = [-1.36 + 0i,     0    + 0i,     0    + 0i,      0    + 0i;
      1.58 - 0.90i, -8.87 + 0i,     0    + 0i,      0    + 0i;
      2.21 + 0.21i, -1.84 + 0.03i, -4.63 + 0i,      0    + 0i;
      3.91 - 1.50i, -1.78 - 1.18i,  0.11 - 0.11i,  -1.84 + 0i];

% Factorize
[af, ipiv, info] = f07mr( ...
                          uplo, a);

% Norm of A
an = a + a' - diag(diag(a));
anorm = norm(an,1);

% Condition number estimator
[rcond, info] = f07mu( ...
                       uplo, af, ipiv, anorm);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

### 9.2 Program Results

```
     f07mu example results

Estimate of condition number =  6.68e+00
```