NAG Toolbox

nag_lapack_dptsv (f07ja)

1 Purpose

nag_lapack_dptsv (f07ja) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric positive definite tridiagonal matrix, and X and B are n by r matrices.

2 Syntax

```
[d, e, b, info] = nag_lapack_dptsv(d, e, b, 'n', n, 'nrhs_p', nrhs_p)
[d, e, b, info] = f07ja(d, e, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dptsv (f07ja) factors A as $A = LDL^{\mathrm{T}}$. The factored form of A is then used to solve the system of equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** must be at least $\max(1, \mathbf{n})$

The n diagonal elements of the tridiagonal matrix A.

2: **e**(:) – REAL (KIND=nag_wp) array

The dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$

The $(n - 1)$ subdiagonal elements of the tridiagonal matrix A.

3: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **b** and the dimension of the array **d**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** will be $\max(1, \mathbf{n})$

The n diagonal elements of the diagonal matrix D from the factorization $A = LDL^{\mathrm{T}}$.

2: **e**(:) – REAL (KIND=nag_wp) array

The dimension of the array **e** will be $\max(1, \mathbf{n} - 1)$

The $(n-1)$ subdiagonal elements of the unit bidiagonal factor L from the LDL^{T} factorization of A. (**e** can also be regarded as the superdiagonal of the unit bidiagonal factor U from the $U^{\mathrm{T}}DU$ factorization of A.)

3: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

If **info** $= 0$, the n by r solution matrix X.

4: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0

The leading minor of order $\langle value \rangle$ is not positive definite, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

nag_lapack_dptsvx (f07jb) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, nag_linsys_real_posdef_tridiag_solve (f04bg) solves $Ax = b$ and returns a forward error bound and condition estimate. nag_linsys_real_posdef_tridiag_solve (f04bg) calls nag_lapack_dptsv (f07ja) to solve the equations.

8 Further Comments

The number of floating-point operations required for the factorization of A is proportional to n, and the number of floating-point operations required for the solution of the equations is proportional to nr, where r is the number of right-hand sides.

The complex analogue of this function is nag_lapack_zptsv (f07jn).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the symmetric positive definite tridiagonal matrix

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 6.0 \\ 9.0 \\ 2.0 \\ 14.0 \\ 7.0 \end{pmatrix}.$$

Details of the LDL^{T} factorization of A are also output.

9.1 Program Text

```
     function f07ja_example

fprintf('f07ja example results\n\n');

% Symmetric tridiagonal A stored as two diagonals
d = [ 4     10     29     25     5];
e = [-2     -6     15     8       ];

% RHS
b = [ 6;
      9;
      2;
     14;
      7];

% Solve
[df, ef, x, info] = f07ja( ...
                           d, e, b);

disp('Solution');
disp(x');
disp('Diagonal elements of the diagonal matrix D');
disp(df);
disp('Sub-diagonal elements of the Cholesky factor L');
disp(ef);
```

9.2 Program Results

```
     f07ja example results

Solution
    2.5000    2.0000    1.0000   -1.0000    3.0000

Diagonal elements of the diagonal matrix D
     4     9    25    16     1

Sub-diagonal elements of the Cholesky factor L
   -0.5000   -0.6667    0.6000    0.5000
```