NAG Toolbox

nag_lapack_dpbcon (f07hg)

1 Purpose

nag_lapack_dpbcon (f07hg) estimates the condition number of a real symmetric positive definite band matrix A, where A has been factorized by nag_lapack_dpbtrf (f07hd).

2 Syntax

```
[rcond, info] = nag_lapack_dpbcon(uplo, kd, ab, anorm, 'n', n)
[rcond, info] = f07hg(uplo, kd, ab, anorm, 'n', n)
```

3 Description

nag_lapack_dpbcon (f07hg) estimates the condition number (in the 1-norm) of a real symmetric positive definite band matrix A:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a computation of $\|A\|_1$ and a call to nag_lapack_dpbtrf (f07hd) to compute the Cholesky factorization of A. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = U^{\mathrm{T}}U$, where U is upper triangular.

uplo = 'L'
$A = LL^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **kd** – INTEGER

k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

3: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **kd** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The Cholesky factor of A, as returned by nag_lapack_dpbtrf (f07hd).

4: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix A. **anorm** must be computed either **before** calling nag_lapack_dpbtrf (f07hd) or else from a **copy** of the original matrix A.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_dpbcon (f07hg) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $4nk$ floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to nag_lapack_dpbtrs (f07he) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is nag_lapack_zpbcon (f07hu).

9 Example

This example estimates the condition number in the 1-norm (or ∞-norm) of the matrix A, where

$$A = \begin{pmatrix} 5.49 & 2.68 & 0.00 & 0.00 \\ 2.68 & 5.63 & -2.39 & 0.00 \\ 0.00 & -2.39 & 2.60 & -2.22 \\ 0.00 & 0.00 & -2.22 & 5.17 \end{pmatrix}.$$

Here A is symmetric and positive definite, and is treated as a band matrix, which must first be factorized by nag_lapack_dpbtrf (f07hd). The true condition number in the 1-norm is 74.15.

9.1 Program Text

```
     function f07hg_example

fprintf('f07hg example results\n\n');

% Symmetric banded matrix A in ab.
uplo = 'L';
kd  = nag_int(1);
ab  = [5.49,  5.63,  2.60, 5.17;
       2.68, -2.39, -2.22, 0.00];

% To calculate 1-norm here, need to add superdiagonal
abn = [0.00,  2.68, -2.39, -2.22;
       ab];
% 1-norm of A = 1-norm of abn
anorm = norm(abn,1);

% Factorize A
[abf, info] = f07hd( ...
                     uplo, kd, ab);

% Get reciprocal condition number
[rcond, info] = f07hg( ...
                       uplo, kd, abf, anorm);

fprintf('Condition number of A = %7.1f\n',1/rcond);
```

9.2 Program Results

```
     f07hg example results

Condition number of A =    74.2
```