NAG Toolbox

nag_lapack_zppcon (f07gu)

1 Purpose

nag_lapack_zppcon (f07gu) estimates the condition number of a complex Hermitian positive definite matrix A, where A has been factorized by nag_lapack_zpptrf (f07gr), using packed storage.

2 Syntax

```
[rcond, info] = nag_lapack_zppcon(uplo, n, ap, anorm)
[rcond, info] = f07gu(uplo, n, ap, anorm)
```

3 Description

nag_lapack_zppcon (f07gu) estimates the condition number (in the 1-norm) of a complex Hermitian positive definite matrix A:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a computation of $\|A\|_1$ and a call to nag_lapack_zpptrf (f07gr) to compute the Cholesky factorization of A. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'
$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – INTEGER

n, the order of the matrix A.

Constraint: **n** ≥ 0.

3: **ap**(:) – COMPLEX (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

The Cholesky factor of A stored in packed form, as returned by nag_lapack_zpptrf (f07gr).

4: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix A. **anorm** must be computed either **before** calling nag_lapack_zpptrf (f07gr) or else from a **copy** of the original matrix A.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_zppcon (f07gu) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to nag_lapack_zpptrs (f07gs) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is nag_lapack_dppcon (f07gg).

9 Example

This example estimates the condition number in the 1-norm (or ∞-norm) of the matrix A, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here A is Hermitian positive definite, stored in packed form, and must first be factorized by nag_lapack_zpptrf (f07gr). The true condition number in the 1-norm is 201.92.

9.1 Program Text

```
     function f07gu_example

fprintf('f07gu example results\n\n');

uplo = 'L';
n = nag_int(4);
ap = [3.23 + 0i    1.51 + 1.92i    1.90 - 0.84i    0.42 - 2.50i ...
                   3.58 + 0i      -0.23 - 1.11i   -1.18 - 1.37i ...
                                   4.09 + 0.00i    2.33 + 0.14i ...
                                                   4.29 + 0.00i];

[L, info] = f07gr( ...
                     uplo, n, ap);

anorm = norm([ap(4) ap(7) ap(9) ap(10)], 1);

[rcond, info] = f07gu( ...
                       uplo, n, L, anorm);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

9.2 Program Results

```
     f07gu example results

Estimate of condition number =  1.51e+02
```