NAG Toolbox

nag_lapack_zpotri (f07fw)

1 Purpose

nag_lapack_zpotri (f07fw) computes the inverse of a complex Hermitian positive definite matrix A, where A has been factorized by nag_lapack_zpotrf (f07fr).

2 Syntax

```
[a, info] = nag_lapack_zpotri(uplo, a, 'n', n)
[a, info] = f07fw(uplo, a, 'n', n)
```

3 Description

nag_lapack_zpotri (f07fw) is used to compute the inverse of a complex Hermitian positive definite matrix A, the function must be preceded by a call to nag_lapack_zpotrf (f07fr), which computes the Cholesky factorization of A.

If **uplo** = 'U', $A = U^{\mathrm{H}}U$ and A^{-1} is computed by first inverting U and then forming $(U^{-1})U^{-\mathrm{H}}$.

If **uplo** = 'L', $A = LL^{\mathrm{H}}$ and A^{-1} is computed by first inverting L and then forming $L^{-\mathrm{H}}(L^{-1})$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'
$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**$(lda, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The upper triangular matrix U if **uplo** = 'U' or the lower triangular matrix L if **uplo** = 'L', as returned by nag_lapack_zpotrf (f07fr).

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{n})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

U stores the upper triangle of A^{-1} if **uplo** = 'U'; L stores the lower triangle of A^{-1} if **uplo** = 'L'.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Diagonal element $\langle value \rangle$ of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n, ϵ is the ***machine precision*** and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2 \left\|A^{-1}\right\|_2.$$

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$.

The real analogue of this function is nag_lapack_dpotri (f07fj).

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here A is Hermitian positive definite and must first be factorized by nag_lapack_zpotrf (f07fr).

9.1 Program Text

```
     function f07fw_example

fprintf('f07fw example results\n\n');

% Lower triangular part of Hermitian matrix A
uplo = 'Lower';
a = [ 3.23 + 0i,     0    + 0i,     0    + 0i,     0    + 0i;
      1.51 + 1.92i,  3.58 + 0i,     0    + 0i,     0    + 0i;
      1.90 - 0.84i, -0.23 - 1.11i,  4.09 + 0i,     0    + 0i;
      0.42 - 2.50i, -1.18 - 1.37i,  2.33 + 0.14i,  4.29 + 0i];

% Factorize
[L, info] = f07fr( ...
                  uplo, a);

% Invert
[ainv, info] = f07fw( ...
                     uplo, L);

[ifail] = x04da( ...
                uplo, 'Non-unit', ainv, 'Inverse');
```

9.2 Program Results

```
     f07fw example results

 Inverse
             1          2          3          4
 1      5.4691
        0.0000

 2     -1.2624     1.1024
       -1.5491     0.0000

 3     -2.9746     0.8989     2.1589
       -0.9616    -0.5672     0.0000

 4      1.1962    -0.9826    -1.3756     2.2934
        2.9772    -0.2566    -1.4550     0.0000
```