

NAG Toolbox

nag_lapack_zpocon (f07fu)

1 Purpose

nag_lapack_zpocon (f07fu) estimates the condition number of a complex Hermitian positive definite matrix A , where A has been factorized by nag_lapack_zpotrf (f07fr).

2 Syntax

```
[rcond, info] = nag_lapack_zpocon(uplo, a, anorm, 'n', n)
[rcond, info] = f07fu(uplo, a, anorm, 'n', n)
```

3 Description

nag_lapack_zpocon (f07fu) estimates the condition number (in the 1-norm) of a complex Hermitian positive definite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a computation of $\|A\|_1$ and a call to nag_lapack_zpotrf (f07fr) to compute the Cholesky factorization of A . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'

$A = U^H U$, where U is upper triangular.

uplo = 'L'

$A = L L^H$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda, :)** – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The Cholesky factor of A , as returned by nag_lapack_zpotrf (f07fr).

3: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix A . **anorm** must be computed either **before** calling `nag_lapack_zpotrf` (f07fr) or else from a **copy** of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to `nag_lapack_zpocon` (f07fu) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to `nag_lapack_zpotrs` (f07fs) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is `nag_lapack_dpocon` (f07fg).

9 Example

This example estimates the condition number in the 1-norm (or ∞ -norm) of the matrix A , where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here A is Hermitian positive definite and must first be factorized by `nag_lapack_zpotrf` (f07fr). The true condition number in the 1-norm is 201.92.

9.1 Program Text

```
function f07fu_example

fprintf('f07fu example results\n\n');

% Hermitian matrix A
uplo = 'L';
a = [ 3.23,          1.51 - 1.92i,  1.90 + 0.84i,  0.42 + 2.50i;
      1.51 + 1.92i,  3.58 + 0i,    -0.23 + 1.11i, -1.18 + 1.37i;
      1.90 - 0.84i, -0.23 - 1.11i,  4.09 + 0i,    2.33 - 0.14i;
      0.42 - 2.50i, -1.18 - 1.37i,  2.33 + 0.14i,  4.29 + 0i];
anorm = norm(a, 1);

% Factorize A
[L, info] = f07fr( ...
              uplo, a);

% Condition number estimator
[rcond, info] = f07fu( ...
                  uplo, L, anorm);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

9.2 Program Results

```
f07fu example results

Estimate of condition number = 1.51e+02
```
