NAG Toolbox

nag_lapack_zpotrs (f07fs)

1 Purpose

nag_lapack_zpotrs (f07fs) solves a complex Hermitian positive definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by nag_lapack_zpotrf (f07fr).

2 Syntax

```
[b, info] = nag_lapack_zpotrs(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07fs(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_zpotrs (f07fs) is used to solve a complex Hermitian positive definite system of linear equations $AX = B$, this function must be preceded by a call to nag_lapack_zpotrf (f07fr) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If **uplo** = 'U', $A = U^{\mathrm{H}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{H}}Y = B$ and then $UX = Y$.

If **uplo** = 'L', $A = LL^{\mathrm{H}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{H}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'
$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**$(lda, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The Cholesky factor of A, as returned by nag_lapack_zpotrf (f07fr).

3: **b**($ldb,:$) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1,\mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **b**($ldb,:$) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1,\mathbf{n})$.

The second dimension of the array **b** will be $\max(1,\mathbf{nrhs\_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon|U^{\mathrm{H}}||U|$;

if **uplo** = 'L', $|E| \leq c(n)\epsilon|L||L^{\mathrm{H}}|$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_zporfs (f07fv), and an estimate for $\kappa_\infty(A)$ $(= \kappa_1(A))$ can be obtained by calling nag_lapack_zpocon (f07fu).

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to nag_lapack_zporfs (f07fv) to refine the solution and return an error estimate.

The real analogue of this function is nag_lapack_dpotrs (f07fe).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3.93 - 6.14i & 1.48 + 6.58i \\ 6.17 + 9.42i & 4.65 - 4.75i \\ -7.17 - 21.83i & -4.91 + 2.29i \\ 1.99 - 14.38i & 7.64 - 10.79i \end{pmatrix}.$$

Here A is Hermitian positive definite and must first be factorized by nag_lapack_zpotrf (f07fr).

9.1 Program Text

```
     function f07fs_example

fprintf('f07fs example results\n\n');

% Lower triangular part of Hermitian matrix A
uplo = 'Lower';
a = [ 3.23 + 0i,     0    + 0i,     0    + 0i,     0    + 0i;
      1.51 + 1.92i,  3.58 + 0i,     0    + 0i,     0    + 0i;
      1.90 - 0.84i, -0.23 - 1.11i,  4.09 + 0i,     0    + 0i;
      0.42 - 2.50i, -1.18 - 1.37i,  2.33 + 0.14i,  4.29 + 0i];

[L, info] = f07fr( ...
                   uplo, a);

% Rhs
b = [ 3.93 -  6.14i,  1.48 +  6.58i;
      6.17 +  9.42i,  4.65 -  4.75i;
     -7.17 - 21.83i, -4.91 +  2.29i;
      1.99 - 14.38i,  7.64 - 10.79i];

% Solve AX = B
[x, info] = f07fs( ...
                   uplo, L, b);

disp('Solution(s)');
disp(x);
```

9.2 Program Results

```
     f07fs example results

Solution(s)
   1.0000 - 1.0000i  -1.0000 + 2.0000i
  -0.0000 + 3.0000i   3.0000 - 4.0000i
  -4.0000 - 5.0000i  -2.0000 + 3.0000i
   2.0000 + 1.0000i   4.0000 - 5.0000i
```