NAG Toolbox

nag_lapack_dpocon (f07fg)

1 Purpose

nag_lapack_dpocon (f07fg) estimates the condition number of a real symmetric positive definite matrix A, where A has been factorized by nag_lapack_dpotrf (f07fd).

2 Syntax

```
[rcond, info] = nag_lapack_dpocon(uplo, a, anorm, 'n', n)
[rcond, info] = f07fg(uplo, a, anorm, 'n', n)
```

3 Description

nag_lapack_dpocon (f07fg) estimates the condition number (in the 1-norm) of a real symmetric positive definite matrix A:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a computation of $\|A\|_1$ and a call to nag_lapack_dpotrf (f07fd) to compute the Cholesky factorization of A. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = U^{\mathrm{T}}U$, where U is upper triangular.

uplo = 'L'
$A = LL^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The Cholesky factor of A, as returned by nag_lapack_dpotrf (f07fd).

3: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix A. **anorm** must be computed either **before** calling nag_lapack_dpotrf (f07fd) or else from a **copy** of the original matrix A.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_dpocon (f07fg) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to nag_lapack_dpotrs (f07fe) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is nag_lapack_zpocon (f07fu).

9 Example

This example estimates the condition number in the 1-norm (or ∞-norm) of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

Here A is symmetric positive definite and must first be factorized by nag_lapack_dpotrf (f07fd). The true condition number in the 1-norm is 97.32.

9.1 Program Text

```
     function f07fg_example

fprintf('f07fg example results\n\n');

a = [ 4.16, -3.12,  0.56, -0.10;
     -3.12,  5.03, -0.83,  1.18;
      0.56, -0.83,  0.76,  0.34;
     -0.10,  1.18,  0.34,  1.18];

% Factorize
uplo = 'L';
[af, info] = f07fd(uplo, a);

% Estimate condition number
anorm = norm(a, 1);
[rcond, info] = f07fg( ...
                       uplo, af, anorm);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

9.2 Program Results

```
     f07fg example results

Estimate of condition number =  9.73e+01
```