NAG Toolbox

nag_lapack_dpoequ (f07ff)

1 Purpose

nag_lapack_dpoequ (f07ff) computes a diagonal scaling matrix S intended to equilibrate a real n by n symmetric positive definite matrix A and reduce its condition number.

2 Syntax

```
[s, scond, amax, info] = nag_lapack_dpoequ(a, 'n', n)
[s, scond, amax, info] = f07ff(a, 'n', n)
```

3 Description

nag_lapack_dpoequ (f07ff) computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B, $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham (2002)).

4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The matrix A whose scaling factors are to be computed. Only the diagonal elements of the array **a** are referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **s**$(\mathbf{n})$ – REAL (KIND=nag_wp) array

If **info** $= 0$, **s** contains the diagonal elements of the scaling matrix S.

2: **scond** – REAL (KIND=nag_wp)

If **info** $= 0$, **scond** contains the ratio of the smallest value of **s** to the largest value of **s**. If **scond** ≥ 0.1 and **amax** is neither too large nor too small, it is not worth scaling by S.

3: **amax** – REAL (KIND=nag_wp)

$\max |a_{ij}|$. If **amax** is very close to overflow or underflow, the matrix A should be scaled.

4: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0

The ⟨*value*⟩th diagonal element of A is not positive (and hence A cannot be positive definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Further Comments

The complex analogue of this function is nag_lapack_zpoequ (f07ft).

9 Example

This example equilibrates the symmetric positive definite matrix A given by

$$A = \begin{pmatrix} 4.16 & -3.12 \times 10^5 & 0.56 & -0.10 \\ -3.12 \times 10^5 & 5.03 \times 10^{10} & -0.83 \times 10^5 & 1.18 \times 10^5 \\ 0.56 & -0.83 \times 10^5 & 0.76 & 0.34 \\ -0.10 & 1.18 \times 10^5 & 0.34 & 1.18 \end{pmatrix}.$$

Details of the scaling factors and the scaled matrix are output.

9.1 Program Text

```
function f07ff_example

fprintf('f07ff example results\n\n');

% Upper triangular part of symmetric matrix A
a = [ 4.16      -3.12e+05   0.56      -0.10    ;
      0          5.03e+10  -8.30e+04   1.18e+05;
      0          0          0.76       0.34    ;
      0          0          0          1.18    ];

% Scale A
[s, scond, amax, info] = f07ff(a);

fprintf('scond = %8.1e, amax = %8.1e\n\n', scond, amax);
disp('Diagonal scaling factors');
fprintf('%10.1e',s);
fprintf('\n\n');
```

```
% Scaled matrix
as = diag(s)*a*diag(s);

[ifail] = x04ca( ...
                 'Upper', 'Non-unit', as, 'Scaled matrix');
```

9.2 Program Results

```
     f07ff example results

scond =  3.9e-06, amax =  5.0e+10

Diagonal scaling factors
   4.9e-01   4.5e-06   1.1e+00   9.2e-01

 Scaled matrix
             1          2          3          4
 1      1.0000    -0.6821     0.3149    -0.0451
 2                 1.0000    -0.4245     0.4843
 3                            1.0000     0.3590
 4                                       1.0000
```