NAG Toolbox

nag_lapack_dpotrs (f07fe)

1 Purpose

nag_lapack_dpotrs (f07fe) solves a real symmetric positive definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by nag_lapack_dpotrf (f07fd).

2 Syntax

```
[b, info] = nag_lapack_dpotrs(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07fe(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dpotrs (f07fe) is used to solve a real symmetric positive definite system of linear equations $AX = B$, this function must be preceded by a call to nag_lapack_dpotrf (f07fd) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If **uplo** $=$ 'U', $A = U^{\mathrm{T}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{T}}Y = B$ and then $UX = Y$.

If **uplo** $=$ 'L', $A = LL^{\mathrm{T}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo $=$ 'U'
$A = U^{\mathrm{T}}U$, where U is upper triangular.

uplo $=$ 'L'
$A = LL^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** $=$ 'U' or 'L'.

2: **a**(lda, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The Cholesky factor of A, as returned by nag_lapack_dpotrf (f07fd).

3: **b**($ldb,:$) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **b**($ldb,:$) – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon|U^{\mathrm{T}}||U|$;

if **uplo** = 'L', $|E| \leq c(n)\epsilon|L||L^{\mathrm{T}}|$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_dporfs (f07fh), and an estimate for $\kappa_\infty(A)$ $(= \kappa_1(A))$ can be obtained by calling nag_lapack_dpocon (f07fg).

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to nag_lapack_dporfs (f07fh) to refine the solution and return an error estimate.

The complex analogue of this function is nag_lapack_zpotrs (f07fs).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 8.70 & 8.30 \\ -13.35 & 2.13 \\ 1.89 & 1.61 \\ -4.14 & 5.00 \end{pmatrix}.$$

Here A is symmetric positive definite and must first be factorized by nag_lapack_dpotrf (f07fd).

9.1 Program Text

```
     function f07fe_example

fprintf('f07fe example results\n\n');

% Lower triangular part of symmetric matrix A
uplo = 'Lower';
a = [ 4.16,  0,     0,    0;
     -3.12,  5.03, 0,    0;
      0.56, -0.83, 0.76, 0;
     -0.10,  1.18, 0.34, 1.18];

% Factorize
[L, info] = f07fd( ...
                   uplo, a);

% Rhs
b = [  8.70, 8.30;
     -13.35, 2.13;
       1.89, 1.61;
      -4.14, 5.00];

% Solve
[x, info] = f07fe( ...
                   uplo, L, b);

disp('Solution(s)');
disp(x);
```

9.2 Program Results

```
     f07fe example results

Solution(s)
    1.0000    4.0000
   -1.0000    3.0000
    2.0000    2.0000
   -3.0000    1.0000
```