NAG Toolbox

nag_lapack_dpotrf (f07fd)

1 Purpose

nag_lapack_dpotrf (f07fd) computes the Cholesky factorization of a real symmetric positive definite matrix.

2 Syntax

```
[a, info] = nag_lapack_dpotrf(uplo, a, 'n', n)
[a, info] = f07fd(uplo, a, 'n', n)
```

3 Description

nag_lapack_dpotrf (f07fd) forms the Cholesky factorization of a real symmetric positive definite matrix A either as $A = U^{\mathrm{T}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{T}}$ if **uplo** = 'L', where U is an upper triangular matrix and L is lower triangular.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville http://www.netlib.org/lapack/lawnspdf/lawn14.pdf

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'
: The upper triangular part of A is stored and A is factorized as $U^{\mathrm{T}}U$, where U is upper triangular.

uplo = 'L'
: The lower triangular part of A is stored and A is factorized as LL^{T}, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n symmetric positive definite matrix A.

If **uplo** = 'U', the upper triangular part of a must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of a must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{n})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The upper or lower triangle of A stores the Cholesky factor U or L as specified by **uplo**.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

$\mathbf{info} > 0$

The leading minor of order $\langle value \rangle$ is not positive definite and the factorization could not be completed. Hence A itself is not positive definite. This may indicate an error in forming the matrix A. To factorize a symmetric matrix which is not positive definite, call nag_lapack_dsytrf (f07md) instead.

7 Accuracy

If $\mathbf{uplo} =$ 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon|U^{\mathrm{T}}||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***. If $\mathbf{uplo} =$ 'L', a similar statement holds for the computed factor L. It follows that $|e_{ij}| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to nag_lapack_dpotrf (f07fd) may be followed by calls to the functions:

nag_lapack_dpotrs (f07fe) to solve $AX = B$;

nag_lapack_dpocon (f07fg) to estimate the condition number of A;

nag_lapack_dpotri (f07fj) to compute the inverse of A.

The complex analogue of this function is nag_lapack_zpotrf (f07fr).

9 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

9.1 Program Text

```
function f07fd_example

fprintf('f07fd example results\n\n');

% Lower triangular part of symmetric matrix A
uplo = 'Lower';
a = [ 4.16,  0,    0,    0;
     -3.12,  5.03, 0,    0;
      0.56, -0.83, 0.76, 0;
     -0.10,  1.18, 0.34, 1.18];

[L, info] = f07fd( ...
                   uplo, a);

[ifail] = x04ca( ...
                 uplo, 'Non-unit', L, 'Cholesky factor');
```

9.2 Program Results

```
f07fd example results

 Cholesky factor
             1          2          3          4
 1      2.0396
 2     -1.5297     1.6401
 3      0.2746    -0.2500     0.7887
 4     -0.0490     0.6737     0.6617     0.5347
```