NAG Toolbox

nag_lapack_dposv (f07fa)

1 Purpose

nag_lapack_dposv (f07fa) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric positive definite matrix and X and B are n by r matrices.

2 Syntax

```
[a, b, info] = nag_lapack_dposv(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
[a, b, info] = f07fa(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dposv (f07fa) uses the Cholesky decomposition to factor A as $A = U^{T}U$ if **uplo** $=$ 'U' or $A = LL^{T}$ if **uplo** $=$ 'L', where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

If **uplo** $=$ 'U', the upper triangle of A is stored.

If **uplo** $=$ 'L', the lower triangle of A is stored.

Constraint: **uplo** $=$ 'U' or 'L'.

2: $\mathbf{a}(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n symmetric matrix A.

If **uplo** $=$ 'U', the upper triangular part of a must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** $=$ 'L', the lower triangular part of a must be stored and the elements of the array above the diagonal are not referenced.

3: $\mathbf{b}(ldb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **b** must be at least max$(1,$**nrhs_p**$)$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the array **a**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: **n** ≥ 0.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: **nrhs_p** ≥ 0.

5.3 Output Parameters

1: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be max$(1,$**n**$)$.

The second dimension of the array **a** will be max$(1,$**n**$)$.

If **info** $= 0$, the factor U or L from the Cholesky factorization $A = U^{\mathrm{T}}U$ or $A = LL^{\mathrm{T}}$.

2: **b**$(ldb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be max$(1,$**n**$)$.

The second dimension of the array **b** will be max$(1,$**nrhs_p**$)$.

If **info** $= 0$, the n by r solution matrix X.

3: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0

The leading minor of order $\langle value \rangle$ of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x}-x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

nag_lapack_dposvx (f07fb) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, nag_linsys_real_posdef_solve (f04bd) solves $Ax = b$ and returns a forward error bound and condition estimate. nag_linsys_real_posdef_solve (f04bd) calls nag_lapack_dposv (f07fa) to solve the equations.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogue of this function is nag_lapack_zposv (f07fn).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the symmetric positive definite matrix

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 8.70 \\ -13.35 \\ 1.89 \\ -4.14 \end{pmatrix}.$$

Details of the Cholesky factorization of A are also output.

9.1 Program Text

```
     function f07fa_example

fprintf('f07fa example results\n\n');

% Upper triangular part of symmetric matrix A
uplo = 'Upper';
a = [4.16, -3.12,  0.56, -0.10;
     0,     5.03, -0.83,  1.18;
     0,     0,     0.76,  0.34;
     0,     0,     0,     1.18];

% RHS
b = [8.7; -13.35;  1.89; -4.14];

% Solve Ax = b
  [af, x, info] = f07fa( ...
                         uplo, a, b);

disp('Solution');
disp(x');

[ifail] = x04ca( ...
                 uplo, 'Non-unit', af, 'Cholesky factor');
```

9.2 Program Results

```
     f07fa example results

Solution
    1.0000   -1.0000    2.0000   -3.0000

 Cholesky factor
            1          2          3          4
 1      2.0396    -1.5297     0.2746    -0.0490
 2                 1.6401    -0.2500     0.6737
 3                            0.7887     0.6617
 4                                       0.5347
```