NAG Toolbox

nag_lapack_dgtsv (f07ca)

1 Purpose

nag_lapack_dgtsv (f07ca) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n tridiagonal matrix and X and B are n by r matrices.

2 Syntax

```
[dl, d, du, b, info] = nag_lapack_dgtsv(dl, d, du, b, 'n', n, 'nrhs_p', nrhs_p)
[dl, d, du, b, info] = f07ca(dl, d, du, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dgtsv (f07ca) uses Gaussian elimination with partial pivoting and row interchanges to solve the equations $AX = B$. The matrix A is factorized as $A = PLU$, where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element per column, and U is an upper triangular band matrix, with two superdiagonals.

Note that the equations $A^{\mathrm{T}}X = B$ may be solved by interchanging the order of the arguments **du** and **dl**.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

5 Parameters

5.1 Compulsory Input Parameters

1: **dl**(:) – REAL (KIND=nag_wp) array

The dimension of the array **dl** must be at least $\max(1, \mathbf{n} - 1)$

Must contain the $(n - 1)$ subdiagonal elements of the matrix A.

2: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** must be at least $\max(1, \mathbf{n})$

Must contain the n diagonal elements of the matrix A.

3: **du**(:) – REAL (KIND=nag_wp) array

The dimension of the array **du** must be at least $\max(1, \mathbf{n} - 1)$

Must contain the $(n - 1)$ superdiagonal elements of the matrix A.

4: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

Note: to solve the equations $Ax = b$, where b is a single right-hand side, **b** may be supplied as a one-dimensional array with length $ldb = \max(1, \mathbf{n})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **b** and the dimension of the array **d**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **dl**(:) – REAL (KIND=nag_wp) array

The dimension of the array **dl** will be $\max(1, \mathbf{n} - 1)$

If no constraints are violated, **dl** stores the $(n - 2)$ elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A, in $\mathbf{dl}(1), \mathbf{dl}(2), \ldots, \mathbf{dl}(n - 2)$.

2: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** will be $\max(1, \mathbf{n})$

If no constraints are violated, **d** stores the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

3: **du**(:) – REAL (KIND=nag_wp) array

The dimension of the array **du** will be $\max(1, \mathbf{n} - 1)$

If no constraints are violated, **du** stores the $(n - 1)$ elements of the first superdiagonal of U.

4: **b**$(ldb, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

Note: to solve the equations $Ax = b$, where b is a single right-hand side, **b** may be supplied as a one-dimensional array with length $ldb = \max(1, \mathbf{n})$.

If $\mathbf{info} = 0$, the n by r solution matrix X.

5: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element $\langle value \rangle$ of the diagonal is exactly zero, and the solution has not been computed. The factorization has not been completed unless $\mathbf{n} = \langle value \rangle$.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Alternatives to nag_lapack_dgtsv (f07ca), which return condition and error estimates are nag_linsys_real_tridiag_solve (f04bc) and nag_lapack_dgtsvx (f07cb).

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr.

The complex analogue of this function is nag_lapack_zgtsv (f07cn).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the tridiagonal matrix

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2.7 \\ -0.5 \\ 2.6 \\ 0.6 \\ 2.7 \end{pmatrix}.$$

9.1 Program Text

```
function f07ca_example

fprintf('f07ca example results\n\n');

% Tridiagonal matrix A stored as diagonals:
du = [        2.1    -1.0      1.9     8.0];
d  = [3.0     2.3    -5.0     -0.9     7.1];
dl = [3.4     3.6     7.0     -6.0         ];

% RHS B
b  = [2.7;   -0.5;    2.6;     0.6;    2.7];

% Solve Ax = B
```

```
[dlf, df, duf, x, info] = f07ca( ...
                                 dl, d, du, b);

disp('Solution');
disp(x');
```

9.2 Program Results

```
     f07ca example results

Solution
   -4.0000    7.0000    3.0000   -4.0000   -3.0000
```