# NAG Toolbox

# nag_lapack_zgbcon (f07bu)

## 1 Purpose

nag_lapack_zgbcon (f07bu) estimates the condition number of a complex band matrix $A$, where $A$ has been factorized by nag_lapack_zgbtrf (f07br).

## 2 Syntax

```
[rcond, info] = nag_lapack_zgbcon(norm_p, kl, ku, ab, ipiv, anorm, 'n', n)
[rcond, info] = f07bu(norm_p, kl, ku, ab, ipiv, anorm, 'n', n)
```

## 3 Description

nag_lapack_zgbcon (f07bu) estimates the condition number of a complex band matrix $A$, in either the 1-norm or the $\infty$-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^{\mathrm{H}})$.

Because the condition number is infinite if $A$ is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function should be preceded by a call to to compute $\|A\|_1$ or $\|A\|_\infty$, and a call to nag_lapack_zgbtrf (f07br) to compute the $LU$ factorization of $A$. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$.

## 4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **norm_p** – CHARACTER(1)

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

**norm_p** = '1' or 'O'
$\kappa_1(A)$ is estimated.

**norm_p** = 'I'
$\kappa_\infty(A)$ is estimated.

*Constraint*: **norm_p** = '1', 'O' or 'I'.

2: **kl** – INTEGER

$k_l$, the number of subdiagonals within the band of the matrix $A$.

*Constraint*: **kl** $\geq 0$.

3: **ku** – INTEGER

$k_u$, the number of superdiagonals within the band of the matrix $A$.

*Constraint*: **ku** $\geq 0$.

4: **ab**$(ldab,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **ab** must be at least $2 \times$ **kl** $+$ **ku** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The $LU$ factorization of $A$, as returned by nag_lapack_zgbtrf (f07br).

5: **ipiv**$(:)$ – INTEGER array

The dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$

The pivot indices, as returned by nag_lapack_zgbtrf (f07br).

6: **anorm** – REAL (KIND=nag_wp)

If **norm_p** $=$ '1' or 'O', the 1-norm of the **original** matrix $A$.

If **norm_p** $=$ 'I', the $\infty$-norm of the **original** matrix $A$.

**anorm** must be computed either **before** calling nag_lapack_zgbtrf (f07br) or else from a **copy** of the original matrix $A$ (see Section 10).

*Constraint*: **anorm** $\geq 0.0$.

### 5.2 Optional Input Parameters

1: **n** – INTEGER

*Default*: the second dimension of the array **ab**.

$n$, the order of the matrix $A$.

*Constraint*: **n** $\geq 0$.

### 5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of $A$. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, $A$ is singular to working precision.

2: **info** – INTEGER

**info** $= 0$ unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

**info** $< 0$

If **info** $= -i$, argument $i$ had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The computed estimate **rcond** is never less than the true value $\rho$, and in practice is nearly always less than $10\rho$, although examples can be constructed where **rcond** is much larger.

## 8 Further Comments

A call to nag_lapack_zgbcon (f07bu) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{H}}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n(2k_l + k_u)$ real floating-point operations (assuming $n \gg k_l$ and $n \gg k_u$) but takes considerably longer than a call to nag_lapack_zgbtrs (f07bs) with one right-hand side, because extra care is taken to avoid overflow when $A$ is approximately singular.

The real analogue of this function is nag_lapack_dgbcon (f07bg).

## 9 Example

This example estimates the condition number in the 1-norm of the matrix $A$, where

$$A = \begin{pmatrix} -1.65 + 2.26i & -2.05 - 0.85i & 0.97 - 2.84i & 0.00 + 0.00i \\ 0.00 + 6.30i & -1.48 - 1.75i & -3.99 + 4.01i & 0.59 - 0.48i \\ 0.00 + 0.00i & -0.77 + 2.83i & -1.06 + 1.94i & 3.33 - 1.04i \\ 0.00 + 0.00i & 0.00 + 0.00i & 4.48 - 1.09i & -0.46 - 1.72i \end{pmatrix}.$$

### 9.1 Program Text

```
     function f07bu_example

fprintf('f07bu example results\n\n');

m = nag_int(4);
kl = nag_int(1);
ku = nag_int(2);
ab =  [0    + 0i,     0    + 0i,     0.97 - 2.84i,  0.59 - 0.48i;
       0    + 0i,    -2.05 - 0.85i, -3.99 + 4.01i,  3.33 - 1.04i;
      -1.65 + 2.26i, -1.48 - 1.75i, -1.06 + 1.94i, -0.46 - 1.72i;
       0    + 6.3i,  -0.77 + 2.83i,  4.48 - 1.09i,  0    + 0i];

norm_p = 'one';
anorm = f16ub( ...
               norm_p, m, kl, ku, ab);

% Fcatorize
abf = [ complex(zeros(kl,m)); ab];
[abf, ipiv, info] = f07br( ...
                           m, kl, ku, abf);

[rcond, info] = f07bu( ...
                       norm_p, kl, ku, abf, ipiv, anorm);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

### 9.2 Program Results

```
     f07bu example results

Estimate of condition number =  1.04e+02
```