NAG Toolbox

nag_lapack_dgbtrf (f07bd)

1 Purpose

nag_lapack_dgbtrf (f07bd) computes the LU factorization of a real m by n band matrix.

2 Syntax

```
[ab, ipiv, info] = nag_lapack_dgbtrf(m, kl, ku, ab, 'n', n)
[ab, ipiv, info] = f07bd(m, kl, ku, ab, 'n', n)
```

3 Description

nag_lapack_dgbtrf (f07bd) forms the LU factorization of a real m by n band matrix A using partial pivoting, with row interchanges. Usually $m = n$, and then, if A has k_l nonzero subdiagonals and k_u nonzero superdiagonals, the factorization has the form $A = PLU$, where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and at most k_l nonzero elements in each column, and U is an upper triangular band matrix with $k_l + k_u$ superdiagonals.

Note that L is not a band matrix, but the nonzero elements of L can be stored in the same space as the subdiagonal elements of A. U is a band matrix but with k_l additional superdiagonals compared with A. These additional superdiagonals are created by the row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **m** – INTEGER

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **kl** – INTEGER

k_l, the number of subdiagonals within the band of the matrix A.

Constraint: $\mathbf{kl} \geq 0$.

3: **ku** – INTEGER

k_u, the number of superdiagonals within the band of the matrix A.

Constraint: $\mathbf{ku} \geq 0$.

4: **ab**$(ldab, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The m by n matrix A.

The matrix is stored in rows k_l+1 to $2k_l+k_u+1$; the first k_l rows need not be set, more precisely, the element A_{ij} must be stored in

$$\mathbf{ab}(k_l+k_u+1+i-j,j) = A_{ij} \quad \text{for } \max(1,j-k_u) \leq i \leq \min(m,j+k_l).$$

See Section 9 in nag_lapack_dgbsv (f07ba) for further details.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** will be $2 \times \mathbf{kl} + \mathbf{ku} + 1$.

The second dimension of the array **ab** will be $\max(1,\mathbf{n})$.

If **info** ≥ 0, **ab** stores details of the factorization.

The upper triangular band matrix U, with k_l+k_u superdiagonals, is stored in rows 1 to k_l+k_u+1 of the array, and the multipliers used to form the matrix L are stored in rows k_l+k_u+2 to $2k_l+k_u+1$.

2: **ipiv**$(\mathbf{min}(\mathbf{m},\mathbf{n}))$ – INTEGER array

The pivot indices that define the permutation matrix. At the ith step, if **ipiv**$(i) > i$ then row i of the matrix A was interchanged with row **ipiv**(i), for $i = 1,2,\ldots,\min(m,n)$. **ipiv**$(i) \leq i$ indicates that, at the ith step, a row interchange was not required.

3: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element $\langle value \rangle$ of the diagonal is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A+E$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l+k_u+1$, and ϵ is the ***machine precision***. This assumes $k \ll \min(m,n)$.

8 Further Comments

The total number of floating-point operations varies between approximately $2nk_l(k_u+1)$ and $2nk_l(k_l+k_u+1)$, depending on the interchanges, assuming $m=n \gg k_l$ and $n \gg k_u$.

A call to nag_lapack_dgbtrf (f07bd) may be followed by calls to the functions:

nag_lapack_dgbtrs (f07be) to solve $AX=B$ or $A^{\mathrm{T}}X=B$;

nag_lapack_dgbcon (f07bg) to estimate the condition number of A.

The complex analogue of this function is nag_lapack_zgbtrf (f07br).

9 Example

This example computes the LU factorization of the matrix A, where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix}.$$

Here A is treated as a band matrix with one subdiagonal and two superdiagonals.

9.1 Program Text

```
function f07bd_example

fprintf('f07bd example results\n\n');

m  = nag_int(4);
kl = nag_int(1);
ku = nag_int(2);
ab = [ 0,    0,     0,     0;
       0,    0,    -3.66, -2.13;
       0,    2.54, -2.73,  4.07;
      -0.23, 2.46,  2.46, -3.82;
      -6.98, 2.56, -4.78,  0];

[abf, ipiv, info] = f07bd( ...
                           m, kl, ku, ab);

mtitle = 'Details of factorization';
[ifail] = x04ce( ...
                 m, m, kl, kl+ku, abf, mtitle);
fprintf('\n');
disp('Pivot indices');
disp(double(ipiv'));
```

9.2 Program Results

```
     f07bd example results

 Details of factorization
            1          2          3          4
 1     -6.9800     2.4600    -2.7300    -2.1300
 2      0.0330     2.5600     2.4600     4.0700
 3                 0.9605    -5.9329    -3.8391
 4                            0.8057    -0.7269

Pivot indices
     2     3     3     4
```