NAG Toolbox

nag_lapack_zgecon (f07au)

1 Purpose

nag_lapack_zgecon (f07au) estimates the condition number of a complex matrix A, where A has been factorized by nag_lapack_zgetrf (f07ar).

2 Syntax

```
[rcond, info] = nag_lapack_zgecon(norm_p, a, anorm, 'n', n)
[rcond, info] = f07au(norm_p, a, anorm, 'n', n)
```

3 Description

nag_lapack_zgecon (f07au) estimates the condition number of a complex matrix A, in either the 1-norm or the ∞-norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^H)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function should be preceded by a computation of $\|A\|_1$ or $\|A\|_\infty$, and a call to nag_lapack_zgetrf (f07ar) to compute the LU factorization of A. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – CHARACTER(1)

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'
$\kappa_1(A)$ is estimated.

norm_p = 'I'
$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The LU factorization of A, as returned by nag_lapack_zgetrf (f07ar).

3: **anorm** – REAL (KIND=nag_wp)

If **norm_p** = '1' or 'O', the 1-norm of the **original** matrix A.

If **norm_p** = 'I', the ∞-norm of the **original** matrix A.

anorm must be computed either **before** calling nag_lapack_zgetrf (f07ar) or else from a **copy** of the original matrix A (see Section 10).

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_zgecon (f07au) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{H}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to nag_lapack_zgetrs (f07as) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is nag_lapack_dgecon (f07ag).

9 Example

This example estimates the condition number in the 1-norm of the matrix A, where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by nag_lapack_zgetrf (f07ar). The true condition number in the 1-norm is 231.86.

9.1 Program Text

```
function f07au_example

fprintf('f07au example results\n\n');

a = [-1.34 + 2.55i,  0.28 + 3.17i, -6.39 - 2.20i,  0.72 - 0.92i;
     -0.17 - 1.41i,  3.31 - 0.15i, -0.15 + 1.34i,  1.29 + 1.38i;
     -3.29 - 2.39i, -1.91 + 4.42i, -0.14 - 1.35i,  1.72 + 1.35i;
      2.41 + 0.39i, -0.56 + 1.47i, -0.83 - 0.69i, -1.96 + 0.67i];

norm_p = '1';
anorm = norm(a, 1);

% Factorise a
[LU, ipiv, info] = f07ar(a);

% Estimate condition number
[rcond, info] = f07au( ...
                       norm_p, LU, anorm);

if rcond > x02aj
  fprintf('\nEstimate of condition number = %10.2e\n', 1/rcond);
else
  fprintf('\nA is singular to working precision\n');
end
```

9.2 Program Results

```
f07au example results

Estimate of condition number =   1.50e+02
```