NAG Toolbox

nag_lapack_zgetrs (f07as)

1 Purpose

nag_lapack_zgetrs (f07as) solves a complex system of linear equations with multiple right-hand sides,

$$AX = B, \quad A^{\mathrm{T}}X = B \quad \text{or} \quad A^{\mathrm{H}}X = B,$$

where A has been factorized by nag_lapack_zgetrf (f07ar).

2 Syntax

```
[b, info] = nag_lapack_zgetrs(trans, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07as(trans, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_zgetrs (f07as) is used to solve a complex system of linear equations $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$, the function must be preceded by a call to nag_lapack_zgetrf (f07ar) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If **trans** = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If **trans** = 'T', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

If **trans** = 'C', the solution is computed by solving $U^{\mathrm{H}}Y = B$ and then $L^{\mathrm{H}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **trans** – CHARACTER(1)

Indicates the form of the equations.

trans = 'N'
$AX = B$ is solved for X.

trans = 'T'
$A^{\mathrm{T}}X = B$ is solved for X.

trans = 'C'
$A^{\mathrm{H}}X = B$ is solved for X.

Constraint: **trans** = 'N', 'T' or 'C'.

2: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The LU factorization of A, as returned by nag_lapack_zgetrf (f07ar).

3: **ipiv**(:) – INTEGER array

The dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$

The pivot indices, as returned by nag_lapack_zgetrf (f07ar).

4: **b**(ldb, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the arrays **a**, **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **b**(ldb, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}(A^{\mathrm{H}})$ (which is the same as $\mathrm{cond}(A^{\mathrm{T}})$) can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_zgerfs (f07av), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_lapack_zgecon (f07au) with **norm_p** = 'I'.

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to nag_lapack_zgerfs (f07av) to refine the solution and return an error estimate.

The real analogue of this function is nag_lapack_dgetrs (f07ae).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -1.34+2.55i & 0.28+3.17i & -6.39-2.20i & 0.72-0.92i \\ -0.17-1.41i & 3.31-0.15i & -0.15+1.34i & 1.29+1.38i \\ -3.29-2.39i & -1.91+4.42i & -0.14-1.35i & 1.72+1.35i \\ 2.41+0.39i & -0.56+1.47i & -0.83-0.69i & -1.96+0.67i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 26.26+51.78i & 31.32-6.70i \\ 6.43-8.68i & 15.86-1.42i \\ -5.75+25.31i & -2.15+30.19i \\ 1.16+2.57i & -2.56+7.55i \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by nag_lapack_zgetrf (f07ar).

9.1 Program Text

```
     function f07as_example

fprintf('f07as example results\n\n');

trans = 'N';
a = [-1.34 + 2.55i,  0.28 + 3.17i, -6.39 - 2.20i,  0.72 - 0.92i;
     -0.17 - 1.41i,  3.31 - 0.15i, -0.15 + 1.34i,  1.29 + 1.38i;
     -3.29 - 2.39i, -1.91 + 4.42i, -0.14 - 1.35i,  1.72 + 1.35i;
      2.41 + 0.39i, -0.56 + 1.47i, -0.83 - 0.69i, -1.96 + 0.67i];
b = [26.26 + 51.78i, 31.32 -  6.70i;
      6.43 -  8.68i, 15.86 -  1.42i;
     -5.75 + 25.31i, -2.15 + 30.19i;
      1.16 +  2.57i, -2.56 +  7.55i];

% Factorize a
[a, ipiv, info] = f07ar(a);

% Compute solution
[x, info] = f07as(trans, a, ipiv, b);

disp('Solution(s)');
fprintf('%11d        ', [1:size(b,2)]);
fprintf('\n');
disp(x);
```

9.2 Program Results

```
     f07as example results

Solution(s)
          1                  2
   1.0000 + 1.0000i  -1.0000 - 2.0000i
   2.0000 - 3.0000i   5.0000 + 1.0000i
  -4.0000 - 5.0000i  -3.0000 + 4.0000i
   0.0000 + 6.0000i   2.0000 - 3.0000i
```