NAG Toolbox

nag_lapack_dgetrs (f07ae)

1 Purpose

nag_lapack_dgetrs (f07ae) solves a real system of linear equations with multiple right-hand sides,

$$AX = B \quad \text{or} \quad A^{\mathrm{T}}X = B,$$

where A has been factorized by nag_lapack_dgetrf (f07ad).

2 Syntax

```
[b, info] = nag_lapack_dgetrs(trans, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07ae(trans, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dgetrs (f07ae) is used to solve a real system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, the function must be preceded by a call to nag_lapack_dgetrf (f07ad) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If **trans** = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If **trans** = 'T' or 'C', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **trans** – CHARACTER(1)

Indicates the form of the equations.

trans = 'N'
$AX = B$ is solved for X.

trans = 'T' or 'C'
$A^{\mathrm{T}}X = B$ is solved for X.

Constraint: **trans** = 'N', 'T' or 'C'.

2: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The LU factorization of A, as returned by nag_lapack_dgetrf (f07ad).

3: **ipiv**$(:)$ – INTEGER array

The dimension of the array **ipiv** must be at least $\max(1,\mathbf{n})$

The pivot indices, as returned by nag_lapack_dgetrf (f07ad).

4: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least max$(1, \mathbf{n})$.

The second dimension of the array **b** must be at least max$(1, \mathbf{nrhs_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the arrays **a**, **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be max$(1, \mathbf{n})$.

The second dimension of the array **b** will be max$(1, \mathbf{nrhs_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}(A^{\mathrm{T}})$ can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_dgerfs (f07ah), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_lapack_dgecon (f07ag) with **norm_p** = 'I'.

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to nag_lapack_dgerfs (f07ah) to refine the solution and return an error estimate.

The complex analogue of this function is nag_lapack_zgetrs (f07as).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 9.52 & 18.47 \\ 24.35 & 2.25 \\ 0.77 & -13.28 \\ -6.22 & -6.21 \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by nag_lapack_dgetrf (f07ad).

9.1 Program Text

```
     function f07ae_example

fprintf('f07ae example results\n\n');

trans = 'N';
a = [1.8, 2.88, 2.05, -0.89;
     5.25, -2.95, -0.95, -3.8;
     1.58, -2.69, -2.9, -1.04;
     -1.11, -0.66, -0.59, 0.8];
b = [9.52, 18.47;
     24.35, 2.25;
     0.77, -13.28;
     -6.22, -6.21];

% Factorize A
[LU, ipiv, info] = f07ad(a);

% Compute Solution
[x, info] = f07ae(trans, LU, ipiv, b);

[ifail] = x04ca( ...
                  'General', ' ', x, 'Solution(s)');
```

9.2 Program Results

```
     f07ae example results

 Solution(s)
             1          2
 1      1.0000     3.0000
 2     -1.0000     2.0000
 3      3.0000     4.0000
 4     -5.0000     1.0000
```