NAG Toolbox

nag_lapack_dgetrf (f07ad)

1 Purpose

nag_lapack_dgetrf (f07ad) computes the LU factorization of a real m by n matrix.

2 Syntax

```
[a, ipiv, info] = nag_lapack_dgetrf(a, 'm', m, 'n', n)
[a, ipiv, info] = f07ad(a, 'm', m, 'n', n)
```

3 Description

nag_lapack_dgetrf (f07ad) forms the LU factorization of a real m by n matrix A as $A = PLU$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). Usually A is square ($m = n$), and both L and U are triangular. The function uses partial pivoting, with row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least max(1, **m**).

The second dimension of the array **a** must be at least max(1, **n**).

The m by n matrix A.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: **m** ≥ 0.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **a**(lda, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be max(1, **m**).

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

2: **ipiv**($\mathbf{min}(\mathbf{m}, \mathbf{n})$) – INTEGER array

The pivot indices that define the permutation matrix. At the ith step, if **ipiv**$(i) > i$ then row i of the matrix A was interchanged with row **ipiv**(i), for $i = 1, 2, \ldots, \min(m, n)$. **ipiv**$(i) \leq i$ indicates that, at the ith step, a row interchange was not required.

3: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element ⟨*value*⟩ of the diagonal is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$ if $m = n$ (the usual case), $\frac{1}{3}n^2(3m - n)$ if $m > n$ and $\frac{1}{3}m^2(3n - m)$ if $m < n$.

A call to this function with $m = n$ may be followed by calls to the functions:

nag_lapack_dgetrs (f07ae) to solve $AX = B$ or $A^{\mathrm{T}}X = B$;

nag_lapack_dgecon (f07ag) to estimate the condition number of A;

nag_lapack_dgetri (f07aj) to compute the inverse of A.

The complex analogue of this function is nag_lapack_zgetrf (f07ar).

9 Example

This example computes the LU factorization of the matrix A, where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix}.$$

9.1 Program Text

```
     function f07ad_example

fprintf('f07ad example results\n\n');

a = [ 1.80,  2.88,  2.05, -0.89;
      5.25, -2.95, -0.95, -3.80;
      1.58, -2.69, -2.90, -1.04;
     -1.11, -0.66, -0.59,  0.80];

% LU Factorize
[LU, ipiv, info] = f07ad(a);

disp('Details of factorization');
disp(LU);
disp('Pivot indices');
disp(double(ipiv'));
```

9.2 Program Results

```
     f07ad example results

Details of factorization
    5.2500   -2.9500   -0.9500   -3.8000
    0.3429    3.8914    2.3757    0.4129
    0.3010   -0.4631   -1.5139    0.2948
   -0.2114   -0.3299    0.0047    0.1314

Pivot indices
     2     2     3     4
```