NAG Toolbox

nag_linsys_real_posdef_solve_1rhs (f04as)

1 Purpose

nag_linsys_real_posdef_solve_1rhs (f04as) calculates the accurate solution of a set of real symmetric positive definite linear equations with a single right-hand side, $Ax = b$, using a Cholesky factorization and iterative refinement.

2 Syntax

```
[a, c, ifail] = nag_linsys_real_posdef_solve_1rhs(a, b, 'n', n)
[a, c, ifail] = f04as(a, b, 'n', n)
```

3 Description

Given a set of real linear equations $Ax = b$, where A is a symmetric positive definite matrix, nag_linsys_real_posdef_solve_1rhs (f04as) first computes a Cholesky factorization of A as $A = LL^{\mathrm{T}}$ where L is lower triangular. An approximation to x is found by forward and backward substitution. The residual vector $r = b - Ax$ is then calculated using ***additional precision*** and a correction d to x is found by solving $LL^{\mathrm{T}}d = r$. x is then replaced by $x + d$, and this iterative refinement of the solution is repeated until machine accuracy is obtained.

4 References

Wilkinson J H and Reinsch C (1971) *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The upper triangle of the n by n positive definite symmetric matrix A. The elements of the array below the diagonal need not be set.

2: **b**$(\mathbf{max}(\mathbf{1}, \mathbf{n}))$ – REAL (KIND=nag_wp) array

The dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The right-hand side vector b.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the arrays **a**, **b**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{n})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

The elements of the array below the diagonal are overwritten; the upper triangle of **a** is unchanged.

2: **c**$(\mathbf{max}(\mathbf{1},\mathbf{n}))$ – REAL (KIND=nag_wp) array

The solution vector x.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The matrix A is not positive definite, possibly due to rounding errors.

ifail $= 2$

Iterative refinement fails to improve the solution, i.e., the matrix A is too ill-conditioned.

ifail $= 3$

On entry, $\mathbf{n} < 0$,
or $lda < \max(1,\mathbf{n})$.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see page 39 of Wilkinson and Reinsch (1971).

8 Further Comments

The time taken by nag_linsys_real_posdef_solve_1rhs (f04as) is approximately proportional to n^3.

The function **must not** be called with the same name for arguments **b** and **c**.

9 Example

This example solves the set of linear equations $Ax = b$ where

$$A = \begin{pmatrix} 5 & 7 & 6 & 5 \\ 7 & 10 & 8 & 7 \\ 6 & 8 & 10 & 9 \\ 5 & 7 & 9 & 10 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 23 \\ 32 \\ 33 \\ 31 \end{pmatrix}.$$

9.1 Program Text

```
     function f04as_example

fprintf('f04as example results\n\n');

% Accurate solution to Ax = b, for positive definite A
a = [5,  7,  6,  5;
     7, 10,  8,  7;
     6,  8, 10,  9;
     5,  7,  9, 10];
b = [23;
     32;
     33;
     31];

[afac, x, ifail] = f04as(a, b);

disp('Solution');
disp(x);
```

9.2 Program Results

```
     f04as example results

Solution
     1
     1
     1
     1
```