NAG Toolbox

nag_matop_real_gen_rq_formq (f01qk)

1 Purpose

nag_matop_real_gen_rq_formq (f01qk) returns the first ℓ rows of the real n by n orthogonal matrix P^{T}, where P is given as the product of Householder transformation matrices.

This function is intended for use following nag_matop_real_gen_rq (f01qj).

2 Syntax

```
[a, ifail] = nag_matop_real_gen_rq_formq(wheret, m, nrowp, a, zeta, 'n', n)
[a, ifail] = f01qk(wheret, m, nrowp, a, zeta, 'n', n)
```

3 Description

P is assumed to be given by

$$P = P_m P_{m-1} \cdots P_1$$

where

$$P_k = I - u_k u_k^{\mathrm{T}},$$

$$u_k = \begin{pmatrix} w_k \\ \zeta_k \\ 0 \\ z_k \end{pmatrix},$$

ζ_k is a scalar, w_k is a $(k-1)$ element vector and z_k is an $(n-m)$ element vector. w_k must be supplied in the kth row of **a** in elements $\mathbf{a}(k,1),\ldots,\mathbf{a}(k,k-1)$. z_k must be supplied in the kth row of **a** in elements $\mathbf{a}(k,m+1),\ldots,\mathbf{a}(k,n)$ and ζ_k must be supplied either in $\mathbf{a}(k,k)$ or in $\mathbf{zeta}(k)$, depending upon the argument **wheret**.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **wheret** – CHARACTER(1)

Indicates where the elements of ζ are to be found.

wheret = 'I' (In **a**)
The elements of ζ are in **a**.

wheret = 'S' (Separate)
The elements of ζ are separate from **a**, in **zeta**.

Constraint: **wheret** = 'I' or 'S'.

2: **m** – INTEGER

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

3: **nrowp** – INTEGER

ℓ, the required number of rows of P.

If $\mathbf{nrowp} = 0$, an immediate return is effected.

Constraint: $0 \leq \mathbf{nrowp} \leq \mathbf{n}$.

4: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m}, \mathbf{nrowp})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The leading m by m strictly lower triangular part of the array **a**, and the m by $(n - m)$ rectangular part of **a** with top left-hand corner at element $\mathbf{a}(1, \mathbf{m} + 1)$ must contain details of the matrix P. In addition, if **wheret** = 'I', the diagonal elements of **a** must contain the elements of ζ.

5: **zeta**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **zeta** must be at least $\max(1, \mathbf{m})$ if **wheret** = 'S', and at least 1 otherwise

With **wheret** = 'S', the array **zeta** must contain the elements of ζ. If $\mathbf{zeta}(k) = 0.0$ then P_k is assumed to be I, otherwise $\mathbf{zeta}(k)$ is assumed to contain ζ_k.

When **wheret** = 'I', the array **zeta** is not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

5.3 Output Parameters

1: **a**$(lda, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m}, \mathbf{nrowp})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The first **nrowp** rows of the array **a** store the first **nrowp** rows of the n by n orthogonal matrix P^{T}.

2: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = −1

On entry, **wheret** ≠ 'I' or 'S',
or **m** < 0,

or **n** $<$ **m**,
or **nrowp** < 0 or **nrowp** $>$ **n**,
or $lda < \max(\mathbf{m}, \mathbf{nrowp})$.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The computed matrix P satisfies the relation

$$P = Q + E,$$

where Q is an exactly orthogonal matrix and

$$\|E\| \leq c\epsilon,$$

ϵ is the ***machine precision*** (see nag_machine_precision (x02aj)), c is a modest function of n, and $\|.\|$ denotes the spectral (two) norm. See also Section 7 in nag_matop_real_gen_rq (f01qj).

8 Further Comments

The approximate number of floating-point operations is given by

$$\begin{array}{ll} \frac{2}{3}m\{(3n-m)(2\ell-m)-m(\ell-m)\}, & \text{if } \ell \geq m, \text{ and} \\ \frac{2}{3}\ell^2(3n-\ell), & \text{if } \ell < m. \end{array}$$

9 Example

This example obtains the 5 by 5 orthogonal matrix P following the RQ factorization of the 3 by 5 matrix A given by

$$A = \begin{pmatrix} 2.0 & 2.0 & 1.6 & 2.0 & 1.2 \\ 2.5 & 2.5 & -0.4 & -0.5 & -0.3 \\ 2.5 & 2.5 & 2.8 & 0.5 & -2.9 \end{pmatrix}.$$

9.1 Program Text

```
function f01qk_example

fprintf('f01qk example results\n\n');

a = [2,   2,    1.6,  2,    1.2;
     2.5, 2.5, -0.4, -0.5, -0.3;
     2.5, 2.5,  2.8,  0.5, -2.9];

[RQ, zeta, ifail] = f01qj(a);

wheret = 'Separate';
m     = nag_int(size(a,1));
nrowp = nag_int(size(a,2));
RQ(m+1:nrowp,1:nrowp) = 0;

[PT, ifail] = f01qk( ...
```

```
                    wheret, m, nrowp, RQ, zeta);

P = PT';
disp('Matrix P');
disp(P);
```

9.2 Program Results

```
     f01qk example results

Matrix P
   -0.1310   -0.5170   -0.4642   -0.5054   -0.4946
   -0.1310   -0.5170   -0.4642    0.5054    0.4946
   -0.3276    0.5499   -0.5199   -0.3957    0.4043
   -0.6551    0.2494   -0.0928    0.4946   -0.5054
   -0.6551   -0.3175    0.5385   -0.2967    0.3032
```