

NAG Toolbox

nag_matop_real_gen_matrix_cond_std (f01ja)

1 Purpose

nag_matop_real_gen_matrix_cond_std (f01ja) computes an estimate of the absolute condition number of a matrix function f at a real n by n matrix A in the 1-norm, where f is either the exponential, logarithm, sine, cosine, hyperbolic sine (sinh) or hyperbolic cosine (cosh). The evaluation of the matrix function, $f(A)$, is also returned.

2 Syntax

```
[a, conda, norma, normfa, ifail] = nag_matop_real_gen_matrix_cond_std(fun, a,
'n', n)
[a, conda, norma, normfa, ifail] = f01ja(fun, a, 'n', n)
```

3 Description

The absolute condition number of f at A , $\text{cond}_{\text{abs}}(f, A)$ is given by the norm of the Fréchet derivative of f , $L(A)$, which is defined by

$$\|L(X)\| := \max_{E \neq 0} \frac{\|L(X, E)\|}{\|E\|},$$

where $L(X, E)$ is the Fréchet derivative in the direction E . $L(X, E)$ is linear in E and can therefore be written as

$$\text{vec}(L(X, E)) = K(X)\text{vec}(E),$$

where the vec operator stacks the columns of a matrix into one vector, so that $K(X)$ is $n^2 \times n^2$. nag_matop_real_gen_matrix_cond_std (f01ja) computes an estimate γ such that $\gamma \leq \|K(X)\|_1$, where $\|K(X)\|_1 \in [n^{-1}\|L(X)\|_1, n\|L(X)\|_1]$. The relative condition number can then be computed via

$$\text{cond}_{\text{rel}}(f, A) = \frac{\text{cond}_{\text{abs}}(f, A)\|A\|_1}{\|f(A)\|_1}.$$

The algorithm used to find γ is detailed in Section 3.4 of Higham (2008).

4 References

Higham N J (2008) *Functions of Matrices: Theory and Computation* SIAM, Philadelphia, PA, USA

5 Parameters

5.1 Compulsory Input Parameters

1: **fun** – CHARACTER(*)

Indicates which matrix function will be used.

fun = 'exp'

The matrix exponential, e^A , will be used.

fun = 'sin'

The matrix sine, $\sin(A)$, will be used.

fun = 'cos'

The matrix cosine, $\cos(A)$, will be used.

fun = 'sinh'

The hyperbolic matrix sine, $\sinh(A)$, will be used.

fun = 'cosh'

The hyperbolic matrix cosine, $\cosh(A)$, will be used.

fun = 'log'

The matrix logarithm, $\log(A)$, will be used.

Constraint: **fun** = 'exp', 'sin', 'cos', 'sinh', 'cosh' or 'log'.

2: **a**(lda,:) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least **n**.

The second dimension of the array **a** must be at least **n**.

The n by n matrix A .

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $n \geq 0$.

5.3 Output Parameters

1: **a**(lda,:) – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be **n**.

The second dimension of the array **a** will be **n**.

The n by n matrix, $f(A)$.

2: **conda** – REAL (KIND=nag_wp)

An estimate of the absolute condition number of f at A .

3: **norma** – REAL (KIND=nag_wp)

The 1-norm of A .

4: **normfa** – REAL (KIND=nag_wp)

The 1-norm of $f(A)$.

5: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

An internal error occurred when evaluating the matrix function $f(A)$. Please contact NAG.

ifail = 2

An internal error occurred when estimating the norm of the Fréchet derivative of f at A . Please contact NAG.

ifail = -1

On entry, **fun** = $\langle value \rangle$ was an illegal value.

ifail = -2

On entry, **n** < 0.

Input argument number $\langle value \rangle$ is invalid.

ifail = -4

On entry, argument *lda* is invalid.

Constraint: $lda \geq \mathbf{n}$.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

`nag_matop_real_gen_matrix_cond_std` (f01ja) uses the norm estimation function `nag_linsys_real_gen_norm_rcomm` (f04yd) to estimate a quantity γ , where $\gamma \leq \|K(\bar{X})\|_1$ and $\|K(X)\|_1 \in [n^{-1}\|L(X)\|_1, n\|L(X)\|_1]$. For further details on the accuracy of norm estimation, see the documentation for `nag_linsys_real_gen_norm_rcomm` (f04yd).

8 Further Comments

The matrix function is computed using one of three underlying matrix function routines:

if **fun** = 'exp', `nag_matop_real_gen_matrix_exp` (f01ec) is used;

if **fun** = 'log', `nag_matop_real_gen_matrix_log` (f01ej) is used;

else, `nag_matop_real_gen_matrix_fun_std` (f01ek) is used.

Approximately $6n^2$ of real allocatable memory is required by the routine, in addition to the memory used by these underlying matrix function routines.

If only $f(A)$ is required, without an estimate of the condition number, then it is far more efficient to use the appropriate matrix function routine listed above.

`nag_matop_complex_gen_matrix_cond_std` (f01ka) can be used to find the condition number of the exponential, logarithm, sine, cosine, sinh or cosh matrix functions at a complex matrix.

9 Example

This example estimates the absolute and relative condition numbers of the matrix sinh function where

$$A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ 3 & -1 & 0 & 2 \\ 1 & 0 & 3 & 1 \\ 1 & 2 & 0 & 3 \end{pmatrix}.$$

9.1 Program Text

```
function f01ja_example

fprintf('f01ja example results\n\n');

a = [2, 1, 3, 1;
     3, -1, 0, 2;
     1, 0, 3, 1;
     1, 2, 0, 3];
fun = 'sinh';

% Find absolute condition number estimate
[a, conda, norma, normfa, ifail] = f01ja( ...
                                     fun, a);

fprintf('\nf(A) = %s(A)\n', fun);
fprintf('Estimated absolute condition number is: %7.2f\n', conda);

% Find relative condition number estimate
eps = x02aj;
if normfa > eps
    cond_rel = conda*norma/normfa;
    fprintf('Estimated relative condition number is: %7.2f\n', cond_rel);
else
    fprintf('The estimated norm of f(A) is effectively zero;\n');
    fprintf('the relative condition number is therefore undefined.\n');
end
```

9.2 Program Results

```
f01ja example results

f(A) = sinh(A)
Estimated absolute condition number is: 204.45
Estimated relative condition number is: 7.90
```
