NAG Toolbox

nag_fit_1dcheb_integ (e02aj)

1 Purpose

nag_fit_1dcheb_integ (e02aj) determines the coefficients in the Chebyshev series representation of the indefinite integral of a polynomial given in Chebyshev series form.

2 Syntax

```
[aintc, ifail] = nag_fit_1dcheb_integ(n, xmin, xmax, a, ia1, qatm1, iaint1)
[aintc, ifail] = e02aj(n, xmin, xmax, a, ia1, qatm1, iaint1)
```

3 Description

nag_fit_1dcheb_integ (e02aj) forms the polynomial which is the indefinite integral of a given polynomial. Both the original polynomial and its integral are represented in Chebyshev series form. If supplied with the coefficients a_i, for $i = 0, 1, \ldots, n$, of a polynomial $p(x)$ of degree n, where

$$p(x) = \tfrac{1}{2}a_0 + a_1T_1(\bar{x}) + \cdots + a_nT_n(\bar{x}),$$

the function returns the coefficients a'_i, for $i = 0, 1, \ldots, n+1$, of the polynomial $q(x)$ of degree $n+1$, where

$$q(x) = \tfrac{1}{2}a'_0 + a'_1T_1(\bar{x}) + \cdots + a'_{n+1}T_{n+1}(\bar{x}),$$

and

$$q(x) = \int p(x)dx.$$

Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree j with argument $\bar{x}$. It is assumed that the normalized variable $\bar{x}$ in the interval $[-1, +1]$ was obtained from your original variable x in the interval $[x_{\mathrm{min}}, x_{\mathrm{max}}]$ by the linear transformation

$$\bar{x} = \frac{2x - (x_{\mathrm{max}} + x_{\mathrm{min}})}{x_{\mathrm{max}} - x_{\mathrm{min}}}$$

and that you require the integral to be with respect to the variable x. If the integral with respect to $\bar{x}$ is required, set $x_{\mathrm{max}} = 1$ and $x_{\mathrm{min}} = -1$.

Values of the integral can subsequently be computed, from the coefficients obtained, by using nag_fit_1dcheb_eval2 (e02ak).

The method employed is that of Chebyshev series (see Chapter 8 of Modern Computing Methods (1961)), modified for integrating with respect to x. Initially taking $a_{n+1} = a_{n+2} = 0$, the function forms successively

$$a'_i = \frac{a_{i-1} - a_{i+1}}{2i} \times \frac{x_{\mathrm{max}} - x_{\mathrm{min}}}{2}, \quad i = n+1, n, \ldots, 1.$$

The constant coefficient a'_0 is chosen so that $q(x)$ is equal to a specified value, **qatm1**, at the lower end point of the interval on which it is defined, i.e., $\bar{x} = -1$, which corresponds to $x = x_{\mathrm{min}}$.

4 References

Modern Computing Methods (1961) Chebyshev-series *NPL Notes on Applied Science* **16** (2nd Edition) HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

n, the degree of the given polynomial $p(x)$.

Constraint: $\mathbf{n} \geq 0$.

2: **xmin** – REAL (KIND=nag_wp)

3: **xmax** – REAL (KIND=nag_wp)

The lower and upper end points respectively of the interval $[x_{\text{min}}, x_{\text{max}}]$. The Chebyshev series representation is in terms of the normalized variable $\bar{x}$, where

$$\bar{x} = \frac{2x - (x_{\text{max}} + x_{\text{min}})}{x_{\text{max}} - x_{\text{min}}}.$$

Constraint: **xmax** > **xmin**.

4: **a**(la) – REAL (KIND=nag_wp) array

la, the dimension of the array, must satisfy the constraint $la \geq 1 + (np1 - 1) \times$ **ia1**.

The Chebyshev coefficients of the polynomial $p(x)$. Specifically, element $i \times$ **ia1** $+ 1$ of **a** must contain the coefficient a_i, for $i = 0, 1, \ldots, n$. Only these $n + 1$ elements will be accessed.

Unchanged on exit, but see **aintc**, below.

5: **ia1** – INTEGER

The index increment of **a**. Most frequently the Chebyshev coefficients are stored in adjacent elements of **a**, and **ia1** must be set to 1. However, if for example, they are stored in **a**(1), **a**(4), **a**$(7), \ldots$, then the value of **ia1** must be 3. See also Section 9.

Constraint: **ia1** ≥ 1.

6: **qatm1** – REAL (KIND=nag_wp)

The value that the integrated polynomial is required to have at the lower end point of its interval of definition, i.e., at $\bar{x} = -1$ which corresponds to $x = x_{\text{min}}$. Thus, **qatm1** is a constant of integration and will normally be set to zero by you.

7: **iaint1** – INTEGER

The index increment of **aintc**. Most frequently the Chebyshev coefficients are required in adjacent elements of **aintc**, and **iaint1** must be set to 1. However, if, for example, they are to be stored in **aintc**(1), **aintc**(4), **aintc**$(7), \ldots$, then the value of **iaint1** must be 3. See also Section 9.

Constraint: **iaint1** ≥ 1.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **aintc**($laint$) – REAL (KIND=nag_wp) array

$laint = 1 + (np1) \times$ **iaint1**.

The Chebyshev coefficients of the integral $q(x)$. (The integration is with respect to the variable x, and the constant coefficient is chosen so that $q(x_{\text{min}})$ equals **qatm1**). Specifically, element $i \times$ **iaint1** $+ 1$ of **aintc** contains the coefficient a'_i, for $i = 0, 1, \ldots, n + 1$. A call of the function may have the array name **aintc** the same as **a**, provided that note is taken of the order in which elements are overwritten when choosing starting elements and increments **ia1** and **iaint1**: i.e., the coefficients, $a_0, a_1, \ldots, a_{i-2}$ must be intact after coefficient a'_i is stored. In particular it is possible to overwrite the a_i entirely by having **ia1** $=$ **iaint1**, and the actual array for **a** and **aintc** identical.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $np1 < 1$,
or **xmax** $\leq$ **xmin**,
or **ia1** < 1,
or $la \leq (np1 - 1) \times$ **ia1**,
or **iaint1** < 1,
or $laint \leq np1 \times$ **iaint1**.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

In general there is a gain in precision in numerical integration, in this case associated with the division by $2i$ in the formula quoted in Section 3.

8 Further Comments

The time taken is approximately proportional to $n + 1$.

The increments **ia1**, **iaint1** are included as arguments to give a degree of flexibility which, for example, allows a polynomial in two variables to be integrated with respect to either variable without rearranging the coefficients.

9 Example

Suppose a polynomial has been computed in Chebyshev series form to fit data over the interval $[-0.5, 2.5]$. The following program evaluates the integral of the polynomial from 0.0 to 2.0. (For the purpose of this example, **xmin**, **xmax** and the Chebyshev coefficients are simply supplied . Normally a program would read in or generate data and compute the fitted polynomial).

9.1 Program Text

```
function e02aj_example

fprintf('e02aj example results\n\n');

% Chebyshev coefficients for f(x) on [-0.5,2.5]
xmin = -0.5; xmax = 2.5;
a = [2.53213  1.13032  0.2715  0.04434   0.00547   0.00054  4e-05];
```

```
n = nag_int(6);
ia1 = nag_int(1);
qatm1 = 0;
iaint1 = nag_int(1);

% Get integral coefficients aint
[aint, ifail] = e02aj( ...
                       n, xmin, xmax, a, ia1, qatm1, iaint1);

% Evaluate integral from x=0 to x=2 by subtraction
xa = 0;
xb = 2;
[inta, ifail] = e02ak( ...
                       n+1, xmin, xmax, aint, ia1, xa);
[intb, ifail] = e02ak( ...
                       n+1, xmin, xmax, aint, ia1, xb);

res = intb - inta;

fprintf('Value of definite integral = %7.4f\n',res);
```

9.2 Program Results

```
     e02aj example results

Value of definite integral =  2.1515
```