NAG Toolbox

nag_ode_bvp_coll_nlin_contin (d02tx)

1 Purpose

nag_ode_bvp_coll_nlin_contin (d02tx) allows a solution to a nonlinear two-point boundary value problem computed by nag_ode_bvp_coll_nlin_solve (d02tl) to be used as an initial approximation in the solution of a related nonlinear two-point boundary value problem in a continuation call to nag_ode_bvp_coll_nlin_solve (d02tl).

2 Syntax

```
[rcomm, icomm, ifail] = nag_ode_bvp_coll_nlin_contin(nmesh, mesh, ipmesh, rcomm, icomm, 'mxmesh', mxmesh)
[rcomm, icomm, ifail] = d02tx(nmesh, mesh, ipmesh, rcomm, icomm, 'mxmesh', mxmesh)
```

3 Description

nag_ode_bvp_coll_nlin_contin (d02tx) and its associated functions (nag_ode_bvp_coll_nlin_solve (d02tl), nag_ode_bvp_coll_nlin_setup (d02tv), nag_ode_bvp_coll_nlin_interp (d02ty) and nag_ode_bvp_coll_nlin_diag (d02tz)) solve the two-point boundary value problem for a nonlinear system of ordinary differential equations

$$\begin{aligned} y_1^{(m_1)}(x) &= f_1\left(x, y_1, y_1^{(1)}, \ldots, y_1^{(m_1-1)}, y_2, \ldots, y_n^{(m_n-1)}\right) \\ y_2^{(m_2)}(x) &= f_2\left(x, y_1, y_1^{(1)}, \ldots, y_1^{(m_1-1)}, y_2, \ldots, y_n^{(m_n-1)}\right) \\ &\vdots \\ y_n^{(m_n)}(x) &= f_n\left(x, y_1, y_1^{(1)}, \ldots, y_1^{(m_1-1)}, y_2, \ldots, y_n^{(m_n-1)}\right) \end{aligned}$$

over an interval $[a, b]$ subject to p (> 0) nonlinear boundary conditions at a and q (> 0) nonlinear boundary conditions at b, where $p + q = \sum_{i=1}^{n} m_i$. Note that $y_i^{(m)}(x)$ is the mth derivative of the ith solution component. Hence $y_i^{(0)}(x) = y_i(x)$. The left boundary conditions at a are defined as

$$g_i(z(y(a))) = 0, \quad i = 1, 2, \ldots, p,$$

and the right boundary conditions at b as

$$\bar{g}_j(z(y(b))) = 0, \quad j = 1, 2, \ldots, q,$$

where $y = (y_1, y_2, \ldots, y_n)$ and

$$z(y(x)) = \left(y_1(x), y_1^{(1)}(x), \ldots, y_1^{(m_1-1)}(x), y_2(x), \ldots, y_n^{(m_n-1)}(x)\right).$$

First, nag_ode_bvp_coll_nlin_setup (d02tv) must be called to specify the initial mesh, error requirements and other details. Then, nag_ode_bvp_coll_nlin_solve (d02tl) can be used to solve the boundary value problem. After successful computation, nag_ode_bvp_coll_nlin_diag (d02tz) can be used to ascertain details about the final mesh. nag_ode_bvp_coll_nlin_interp (d02ty) can be used to compute the approximate solution anywhere on the interval $[a, b]$ using interpolation.

If the boundary value problem being solved is one of a sequence of related problems, for example as part of some continuation process, then nag_ode_bvp_coll_nlin_contin (d02tx) should be used between calls to nag_ode_bvp_coll_nlin_solve (d02tl). This avoids the overhead of a complete initialization when the setup function nag_ode_bvp_coll_nlin_setup (d02tv) is used. nag_ode_bvp_coll_nlin_contin (d02tx) allows the solution values computed in the previous call to nag_ode_bvp_coll_nlin_solve

(d02tl) to be used as an initial approximation for the solution in the next call to nag_ode_bvp_coll_nlin_solve (d02tl).

You must specify the new initial mesh. The previous mesh can be obtained by a call to nag_ode_bvp_coll_nlin_diag (d02tz). It may be used unchanged as the new mesh, in which case any fixed points in the previous mesh remain as fixed points in the new mesh. Fixed and other points may be added or subtracted from the mesh by manipulation of the contents of the array argument **ipmesh**. Initial values for the solution components on the new mesh are computed by interpolation on the values for the solution components on the previous mesh.

The functions are based on modified versions of the codes COLSYS and COLNEW (see Ascher *et al.* (1979) and Ascher and Bader (1987)). A comprehensive treatment of the numerical solution of boundary value problems can be found in Ascher *et al.* (1988) and Keller (1992).

4 References

Ascher U M and Bader G (1987) A new basis implementation for a mixed order boundary value ODE solver *SIAM J. Sci. Stat. Comput.* **8** 483–500

Ascher U M, Christiansen J and Russell R D (1979) A collocation solver for mixed order systems of boundary value problems *Math. Comput.* **33** 659–679

Ascher U M, Mattheij R M M and Russell R D (1988) *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations* Prentice–Hall

Keller H B (1992) *Numerical Methods for Two-point Boundary-value Problems* Dover, New York

5 Parameters

5.1 Compulsory Input Parameters

1: **nmesh** – INTEGER

Suggested value: $(n^* + 1)/2$, where n^* is the number of mesh points used in the previous mesh as returned in the argument **nmesh** of nag_ode_bvp_coll_nlin_diag (d02tz).

The number of points to be used in the new initial mesh. It is strongly recommended that if this function is called that the suggested value (see below) for **nmesh** is used. In this case the arrays **mesh** and **ipmesh** returned by nag_ode_bvp_coll_nlin_diag (d02tz) can be passed to this function without any modification.

Constraint: $6 \leq \mathbf{nmesh} \leq (\mathbf{mxmesh} + 1)/2$.

2: **mesh**(**mxmesh**) – REAL (KIND=nag_wp) array

Suggested value: the argument **mesh** returned from a call to nag_ode_bvp_coll_nlin_diag (d02tz).

The **nmesh** points to be used in the new initial mesh as specified by **ipmesh**.

Constraint: $\mathbf{mesh}(i_j) < \mathbf{mesh}(i_{j+1})$, for $j = 1, 2, \ldots, \mathbf{nmesh} - 1$, the values of $i_1, i_2, \ldots, i_{\mathbf{nmesh}}$ are defined in **ipmesh**.

$\mathbf{mesh}(i_1)$ must contain the left boundary point, a, and $\mathbf{mesh}(i_{\mathbf{nmesh}})$ must contain the right boundary point, b, as specified in the previous call to nag_ode_bvp_coll_nlin_setup (d02tv).

3: **ipmesh**(**mxmesh**) – INTEGER array

Suggested value: the argument **ipmesh** returned in a call to nag_ode_bvp_coll_nlin_diag (d02tz).

Specifies the points in **mesh** to be used as the new initial mesh. Let $\{i_j : j = 1, 2, \ldots, \mathbf{nmesh}\}$ be the set of array indices of **ipmesh** such that $\mathbf{ipmesh}(i_j) = 1$ or 2 and $1 = i_1 < i_2 < \cdots < i_{\mathbf{nmesh}}$. Then $\mathbf{mesh}(i_j)$ will be included in the new initial mesh.

If $\mathbf{ipmesh}(i_j) = 1$, $\mathbf{mesh}(i_j)$ will be a fixed point in the new initial mesh.

If $\mathbf{ipmesh}(k) = 3$ for any k, then $\mathbf{mesh}(k)$ will not be included in the new mesh.

Constraints:

ipmesh$(k) = 1$, 2 or 3, for $k = 1, 2, \ldots, i_{\mathbf{nmesh}}$;
ipmesh$(1) =$ **ipmesh**$(i_{\mathbf{nmesh}}) = 1$.

4: **rcomm**$(*)$ – REAL (KIND=nag_wp) array

Note: the dimension of this array is dictated by the requirements of associated functions that must have been previously called. This array **must** be the same array passed as argument **rcomm** in the previous call to nag_ode_bvp_coll_nlin_solve (d02tl).

This must be the same array as supplied to nag_ode_bvp_coll_nlin_solve (d02tl) and **must** remain unchanged between calls.

5: **icomm**$(*)$ – INTEGER array

Note: the dimension of this array is dictated by the requirements of associated functions that must have been previously called. This array **must** be the same array passed as argument **icomm** in the previous call to nag_ode_bvp_coll_nlin_solve (d02tl).

This must be the same array as supplied to nag_ode_bvp_coll_nlin_solve (d02tl) and **must** remain unchanged between calls.

5.2 Optional Input Parameters

1: **mxmesh** – INTEGER

Default: the dimension of the arrays **mesh**, **ipmesh**. (An error is raised if these dimensions are not equal.)

The maximum number of points allowed in the mesh.

Constraint: this must be identical to the value supplied for the argument **mxmesh** in the prior call to nag_ode_bvp_coll_nlin_setup (d02tv).

5.3 Output Parameters

1: **rcomm**$(*)$ – REAL (KIND=nag_wp) array

Note: the dimension of this array is dictated by the requirements of associated functions that must have been previously called. This array **must** be the same array passed as argument **rcomm** in the previous call to nag_ode_bvp_coll_nlin_solve (d02tl).

Contains information about the solution for use on subsequent calls to associated functions.

2: **icomm**$(*)$ – INTEGER array

Note: the dimension of this array is dictated by the requirements of associated functions that must have been previously called. This array **must** be the same array passed as argument **icomm** in the previous call to nag_ode_bvp_coll_nlin_solve (d02tl).

Contains information about the solution for use on subsequent calls to associated functions.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

An element of **ipmesh** was set to -1 before **nmesh** elements containing 1 or 2 were detected.

Constraint: **ipmesh**$(1) = 1$.

Constraint: **mxmesh** $=$ **mxmesh** in nag_ode_bvp_coll_nlin_setup (d02tv).

Constraint: **nmesh** $\leq$ (**mxmesh** $+ 1)/2$.

Constraint: **nmesh** ≥ 6.

Expected $\langle value \rangle$ elements of **ipmesh** to be 1 or 2, but $\langle value \rangle$ such elements found.

ipmesh$(i) \neq -1$, 1, 2 or 3 for some i.

The entries in **mesh** are not strictly increasing.

The first element of array **mesh** does not coincide with the left hand end of the range previously specified.

The last point of the new mesh does not coincide with the right hand end of the range previously specified.

The solver function did not produce any results suitable for remeshing.

The solver function does not appear to have been called.

You have set the element of **ipmesh** corresponding to the last element of **mesh** to be included in the new mesh as $\langle value \rangle$, which is not 1.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

For problems where sharp changes of behaviour are expected over short intervals it may be advisable to:

– cluster the mesh points where sharp changes in behaviour are expected;

– maintain fixed points in the mesh using the argument **ipmesh** to ensure that the remeshing process does not inadvertently remove mesh points from areas of known interest.

In the absence of any other information about the expected behaviour of the solution, using the values suggested in Section 5 for **nmesh**, **ipmesh** and **mesh** is strongly recommended.

9 Example

This example illustrates the use of continuation, solution on an infinite range, and solution of a system of two differential equations of orders 3 and 2. See also nag_ode_bvp_coll_nlin_solve (d02tl), nag_ode_bvp_coll_nlin_setup (d02tv), nag_ode_bvp_coll_nlin_interp (d02ty) and nag_ode_bvp_coll_nlin_diag (d02tz), for the illustration of other facilities.

Consider the problem of swirling flow over an infinite stationary disk with a magnetic field along the axis of rotation. See Ascher *et al.* (1988) and the references therein. After transforming from a cylindrical coordinate system (r, θ, z), in which the θ component of the corresponding velocity field behaves like r^{-n}, the governing equations are

$$\begin{aligned} f''' + \tfrac{1}{2}(3-n)ff'' + n(f')^2 + g^2 - sf' &= \gamma^2 \\ g'' + \tfrac{1}{2}(3-n)fg' + (n-1)gf' - s(g-1) &= 0 \end{aligned}$$

with boundary conditions

$$f(0) = f'(0) = g(0) = 0, \quad f'(\infty) = 0, \quad g(\infty) = \gamma,$$

where s is the magnetic field strength, and γ is the Rossby number.

Some solutions of interest are for $\gamma = 1$, small n and $s \to 0$. An added complication is the infinite range, which we approximate by $[0, L]$. We choose $n = 0.2$ and first solve for $L = 60.0, s = 0.24$ using the initial approximations $f(x) = -x^2e^{-x}$ and $g(x) = 1.0 - e^{-x}$, which satisfy the boundary conditions, on a uniform mesh of 21 points. Simple continuation on the parameters L and s using the values $L = 120.0, s = 0.144$ and then $L = 240.0, s = 0.0864$ is used to compute further solutions. We use the suggested values for **nmesh**, **ipmesh** and **mesh** in the call to nag_ode_bvp_coll_nlin_contin (d02tx) prior to a continuation call, that is only every second point of the preceding mesh is used.

The equations are first mapped onto $[0, 1]$ to yield

$$\begin{aligned} f''' &= L^3\left(\gamma^2 - g^2\right) + L^2 sg' - L\left(\tfrac{1}{2}(3-n)ff'' + n(g')^2\right) \\ g'' &= L^2 s(g-1) - L\left(\tfrac{1}{2}(3-n)fg' + (n-1)f'g\right). \end{aligned}$$

9.1 Program Text

```
function d02tx_example

fprintf('d02tx example results\n\n');

global en s el; % For communication with local functions

% Initialize variables and arrays.
neq  = nag_int(2);
nlbc = nag_int(3);
nrbc = nag_int(2);
ncol = nag_int(6);
mmax = nag_int(3);
m    = nag_int([3; 2]);
tols = [0.00001; 0.00001];

% Set values for problem-specific physical parameters.
el = 60;
s  = 0.24;
en = 0.2;

% Set up the mesh.
nmesh  = nag_int(21);
mxmesh = nag_int(250);
mesh = zeros(mxmesh,1);
ipmesh = zeros(mxmesh,1,nag_int_name);

% Set initial mesh with constant spacing.
dx = 1/double(nmesh-1);
mesh(1:nmesh) = [0:dx:1];

% Specify mesh end points as fixed.
ipmesh(1)         = 1;
ipmesh(2:nmesh-1) = 2;
ipmesh(nmesh)     = 1;

% Prepare to store results for plotting.
xarray = zeros(1,1);
yarray1 = zeros(1,1);
yarray2 = zeros(1,1);

% d02tv is a setup routine to be called prior to d02tk.
```

```
[work, iwork, ifail] = d02tv(...
                             m, nlbc, nrbc, ncol, tols, nmesh, mesh, ipmesh);

ncont = 3;

% We run through the calculation three times with different parameter sets.
for jcont = 1:ncont
  fprintf('\n Tolerance = %8.1e, L = %8.3f, S = %6.4f\n\n', tols(1), el, s);

  % Call d02tk to solve BVP for this set of parameters.
  [work, iwork, ifail] = d02tk(...
                               @ffun, @fjac, @gafun, @gbfun, @gajac, ...
                               @gbjac, @guess, work, iwork);

  % Call d02tz to extract mesh from solution.
  [nmesh, mesh, ipmesh, ermx, iermx, ijermx, ifail] = ...
  d02tz( ...
        mxmesh, work, iwork);

  % Output mesh results.
  fprintf(' Used a mesh of %d points\n', nmesh);
  fprintf(' Maximum error = %10.2e in interval %d for component %d\n\n',...
          ermx, iermx, ijermx);

  % Output solution, and store it for plotting.
  fprintf(' Solution on original interval:\n');
  fprintf('    x          f           g\n');
  for i=1:16
    xx = double(i-1)*2/el;

    % Call d02ty to perform interpolation on the solution.
    [y, work, ifail] = d02ty(...
                             xx, neq, mmax, work, iwork);
    fprintf(' %6.2f    %8.4f    %8.4f\n', xx*el, y(1,1), y(2,1));
    xarray(i) = xx*el;
    yarray1(i) = y(1,1);
    yarray2(i) = y(2,1);
  end
  for i=1:10
    xx = (30+(el-30)*double(i)/10)/el;

    [y, work, ifail] = d02ty(...
                            xx, neq, mmax, work, iwork);
    fprintf(' %6.2f    %8.4f    %8.4f\n', xx*el, y(1,1), y(2,1));
    xarray(i+16) = xx*el;
    yarray1(i+16) = y(1,1);
    yarray2(i+16) = y(2,1);
  end

  % Plot results for this parameter set.
  if (jcont==1)
    fig1 = figure;
  elseif (jcont==2)
    fig2 = figure;
  else
    fig3 = figure;
  end
  display_plot(xarray, yarray1, yarray2, el, s)

  % Select mesh for next calculation.
  if jcont < ncont
    el = 2*el;
    s = 0.6*s;
    nmesh = (nmesh+1)/2;

    % d02tx allows the current solution to be used as an initial
    % approximation to the solution of a related problem.
    [work, iwork, ifail] = d02tx(...
                                 nmesh, mesh, ipmesh, work, iwork);
  end
end
```

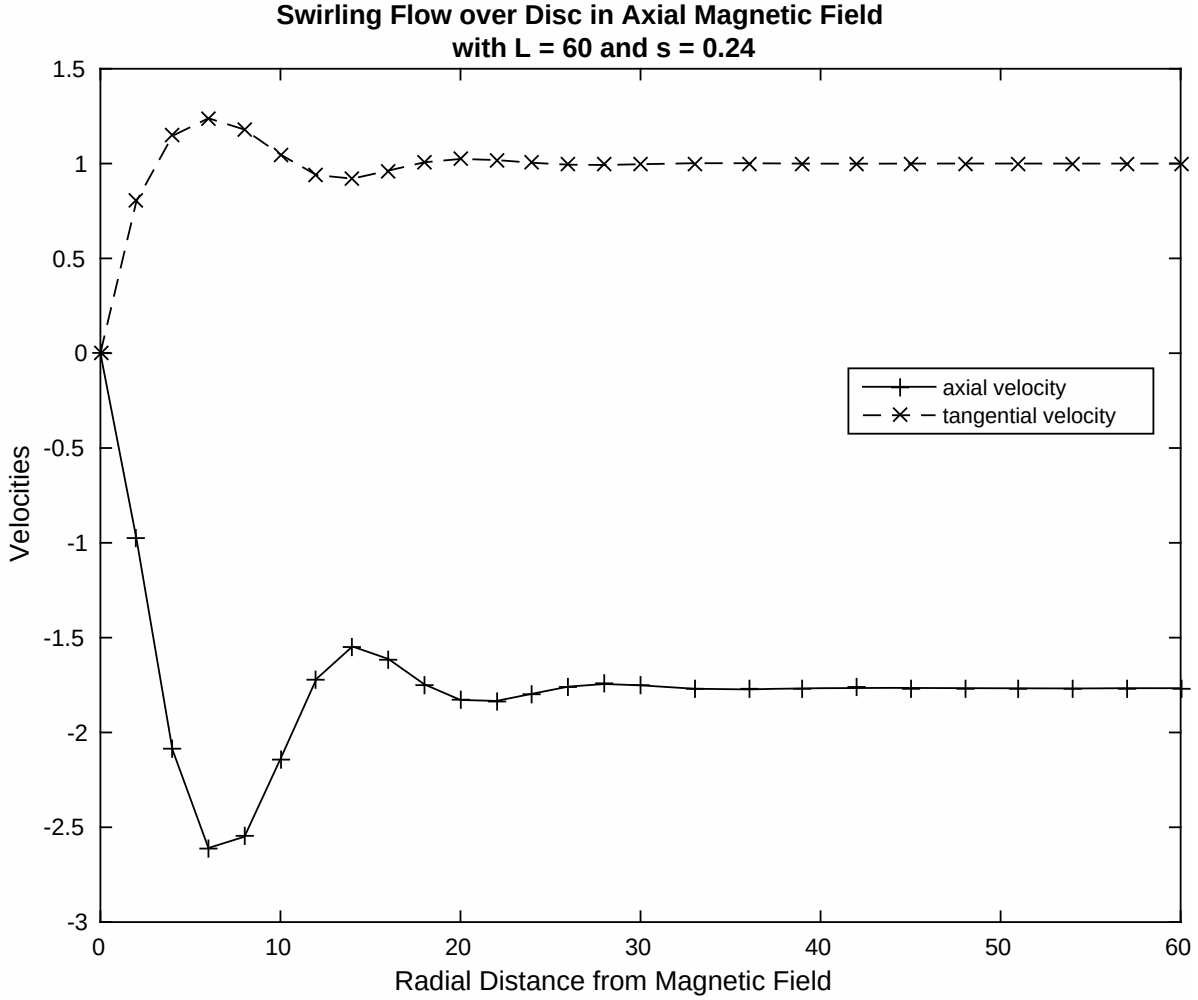
```
function [f] = ffun(x, y, neq, m)
  % Evaluate derivative functions (rhs of system of ODEs).

  global en s el
  f = zeros(neq, 1);
  f(1) = el^3*(1-y(2,1)^2) + el^2*s*y(1,2) - ...
         el*(0.5*(3-en)*y(1,1)*y(1,3) + en*y(1,2)^2);
  f(2) = el^2*s*(y(2,1)-1) - ...
         el*(0.5*(3-en)*y(1,1)*y(2,2) + (en-1)*y(1,2)*y(2,1));

function [dfdy] = fjac(x, y, neq, m)
  % Evaluate Jacobians (partial derivatives of f).

  global en s el
  dfdy = zeros(neq, neq, 3);
  dfdy(1,2,1) = -2*el^3*y(2,1);
  dfdy(1,1,1) = -el*0.5*(3-en)*y(1,3);
  dfdy(1,1,2) = el^2*s - el*2*en*y(1,2);
  dfdy(1,1,3) = -el*0.5*(3-en)*y(1,1);
  dfdy(2,2,1) = el^2*s - el*(en-1)*y(1,2);
  dfdy(2,2,2) = -el*0.5*(3-en)*y(1,1);
  dfdy(2,1,1) = -el*0.5*(3-en)*y(2,2);
  dfdy(2,1,2) = -el*(en-1)*y(2,1);

function [ga] = gafun(ya, neq, m, nlbc)
  % Evaluate boundary conditions at left-hand end of range.

  global en s el
  ga = zeros(nlbc, 1);
  ga(1) = ya(1,1);
  ga(2) = ya(1,2);
  ga(3) = ya(2,1);

function [dgady] = gajac(ya, neq, m, nlbc)
  % Evaluate Jacobians (partial derivatives of ga).

  global en s el
  dgady = zeros(nlbc, neq, 3);
  dgady(1,1,1) = 1;
  dgady(2,1,2) = 1;
  dgady(3,2,1) = 1;

function [gb] = gbfun(yb, neq, m, nrbc)
  % Evaluate boundary conditions at right-hand end of range.

  global en s el
  gb = zeros(nrbc, 1);
  gb(1) = yb(1,2);
  gb(2) = yb(2,1) - 1;

function [dgbdy] = gbjac(yb, neq, m, nrbc)
  % Evaluate Jacobians (partial derivatives of gb).

  global en s el
  dgbdy = zeros(nrbc, neq, 3);
  dgbdy(1,1,2) = 1;
  dgbdy(2,2,1) = 1;

function [y, dym] = guess(x, neq, m)
  % Evaluate initial approximations to solution components and derivatives.

  global en s el
  y = zeros(neq, 3);
  dym = zeros(neq, 1);
  ex = x*el;
  expmx = exp(-ex);
  y(1,1) = -ex^2*expmx;
  y(1,2) = (-2*ex+ex^2)*expmx;
  y(1,3) = (-2+4*ex-ex^2)*expmx;
  y(2,1) = 1 - expmx;
```

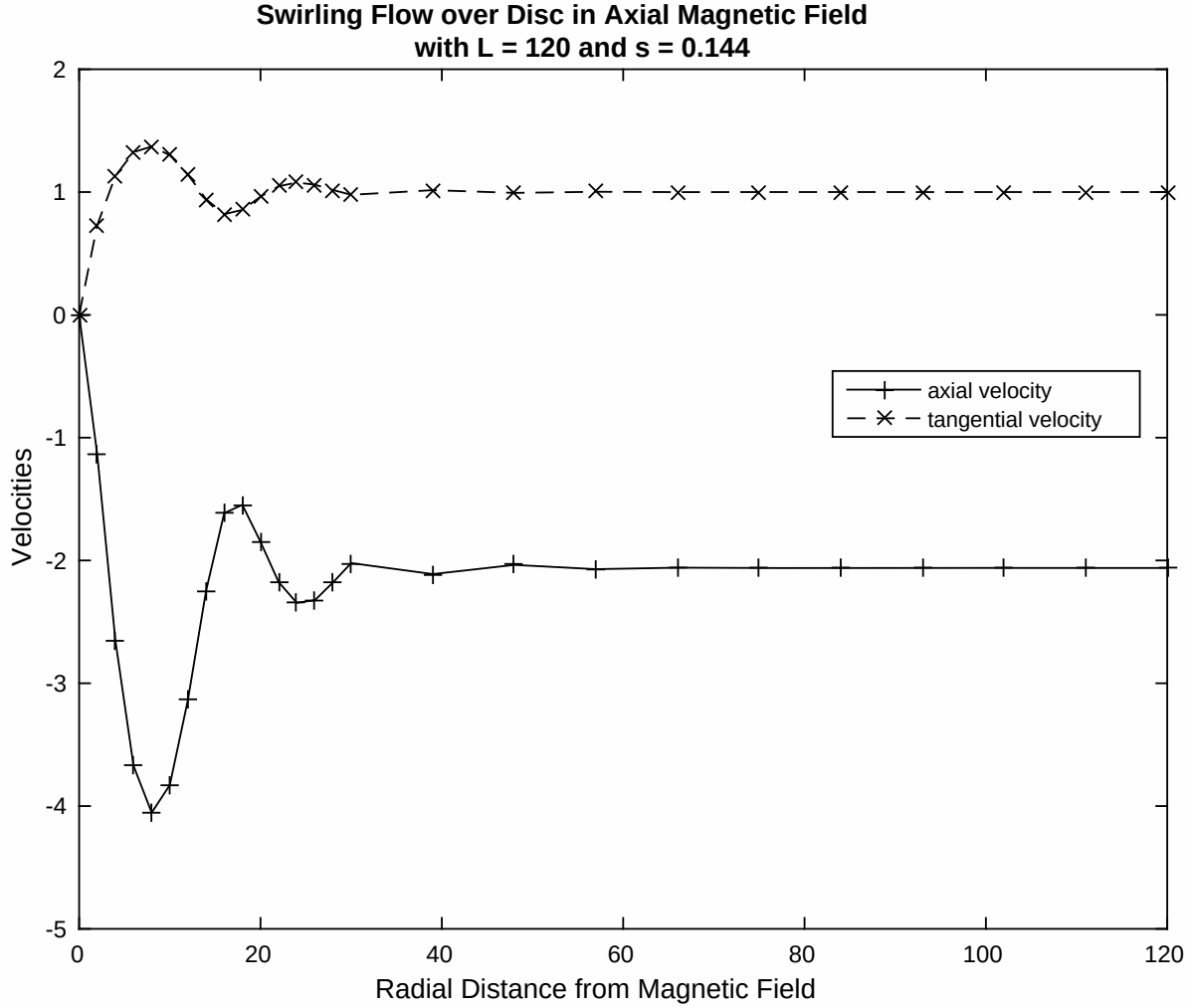
```
  y(2,2) = expmx;
  dym(1) = (6-6*ex+ex^2)*expmx;
  dym(2) = -expmx;

function display_plot(xarray,yarray1,yarray2,el,s)
  % Formatting for title and axis labels.
  % Plot the results.
  plot(xarray, yarray1, '-+', xarray, yarray2, '--x');
  % Add title.
  title({['Swirling Flow over Disc in Axial Magnetic Field '], ...
         ['with L = ',num2str(el), ' and s = ',num2str(s)]})
  % Label the axes.
  xlabel('Radial Distance from Magnetic Field');
  ylabel('Velocities');
  % Add a legend.
  legend('axial velocity','tangential velocity','Location','Best')
```

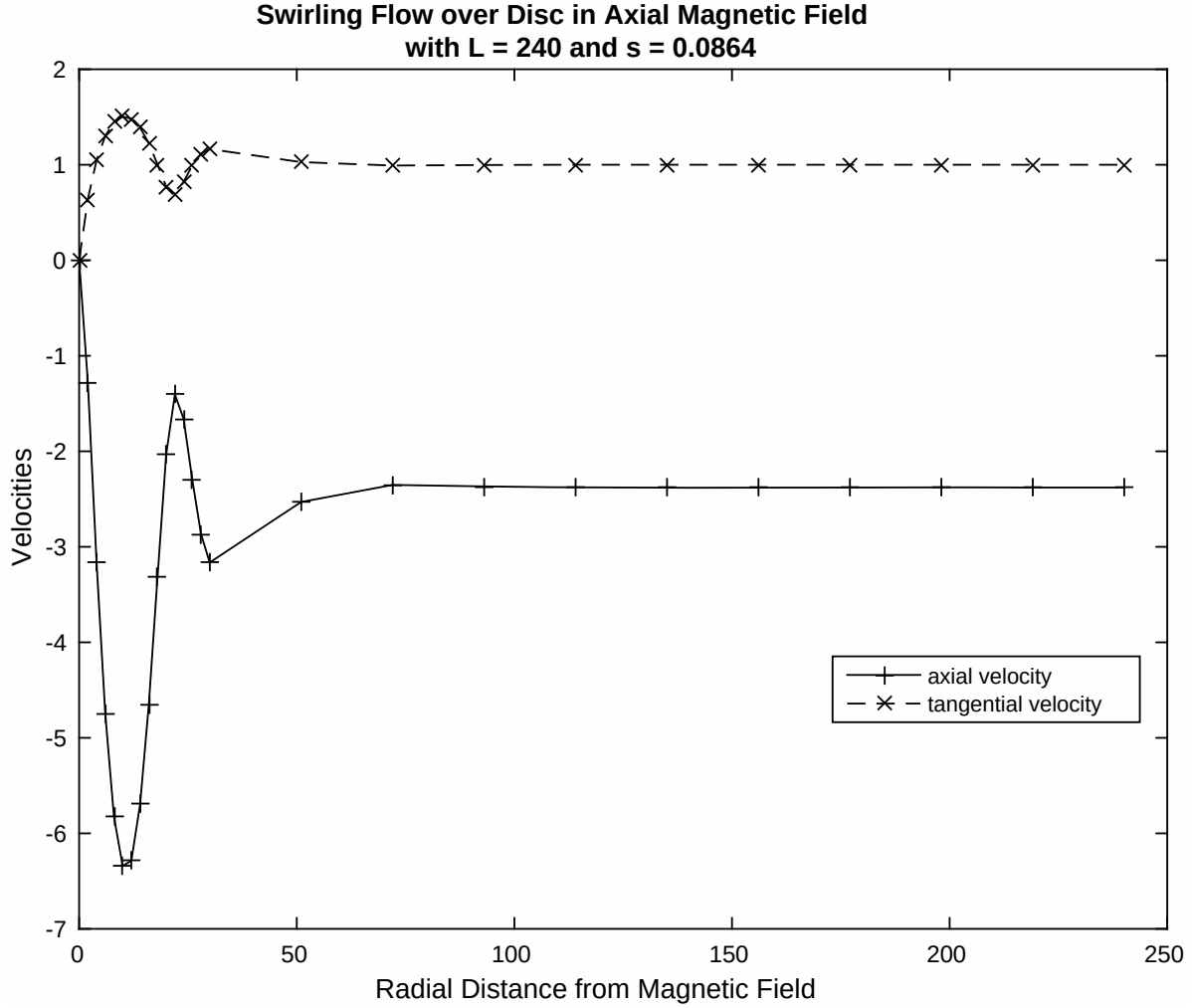
9.2 Program Results

```
     d02tx example results

 Tolerance =  1.0e-05, L =   60.000, S = 0.2400

 Used a mesh of 21 points
 Maximum error =   2.66e-08 in interval 7 for component 1

 Solution on original interval:
    x          f           g
   0.00      0.0000      0.0000
   2.00     -0.9769      0.8011
   4.00     -2.0900      1.1459
   6.00     -2.6093      1.2389
   8.00     -2.5498      1.1794
  10.00     -2.1397      1.0478
  12.00     -1.7176      0.9395
  14.00     -1.5465      0.9206
  16.00     -1.6127      0.9630
  18.00     -1.7466      1.0068
  20.00     -1.8286      1.0244
  22.00     -1.8338      1.0185
  24.00     -1.7956      1.0041
  26.00     -1.7582      0.9940
  28.00     -1.7445      0.9926
  30.00     -1.7515      0.9965
  33.00     -1.7695      1.0019
  36.00     -1.7730      1.0018
  39.00     -1.7673      0.9998
  42.00     -1.7645      0.9993
  45.00     -1.7659      0.9999
  48.00     -1.7672      1.0002
  51.00     -1.7671      1.0001
  54.00     -1.7666      0.9999
  57.00     -1.7665      0.9999
  60.00     -1.7666      1.0000

 Tolerance =  1.0e-05, L =  120.000, S = 0.1440

 Used a mesh of 21 points
 Maximum error =   6.88e-06 in interval 7 for component 2

 Solution on original interval:
    x          f           g
   0.00      0.0000      0.0000
   2.00     -1.1406      0.7317
   4.00     -2.6531      1.1315
   6.00     -3.6721      1.3250
   8.00     -4.0539      1.3707
  10.00     -3.8285      1.3003
  12.00     -3.1339      1.1407
  14.00     -2.2469      0.9424
```

```
 16.00     -1.6146      0.8201
 18.00     -1.5472      0.8549
 20.00     -1.8483      0.9623
 22.00     -2.1761      1.0471
 24.00     -2.3451      1.0778
 26.00     -2.3236      1.0600
 28.00     -2.1784      1.0165
 30.00     -2.0214      0.9775
 39.00     -2.1109      1.0155
 48.00     -2.0362      0.9931
 57.00     -2.0709      1.0023
 66.00     -2.0588      0.9995
 75.00     -2.0616      1.0000
 84.00     -2.0615      1.0001
 93.00     -2.0611      0.9999
102.00     -2.0614      1.0000
111.00     -2.0613      1.0000
120.00     -2.0613      1.0000

Tolerance =  1.0e-05, L =  240.000, S = 0.0864

Used a mesh of 81 points
Maximum error =   3.30e-07 in interval 19 for component 2

Solution on original interval:
   x          f          g
  0.00      0.0000      0.0000
  2.00     -1.2756      0.6404
  4.00     -3.1604      1.0463
  6.00     -4.7459      1.3011
  8.00     -5.8265      1.4467
 10.00     -6.3412      1.5036
 12.00     -6.2862      1.4824
 14.00     -5.6976      1.3886
 16.00     -4.6568      1.2263
 18.00     -3.3226      1.0042
 20.00     -2.0328      0.7718
 22.00     -1.4035      0.6943
 24.00     -1.6603      0.8218
 26.00     -2.2975      0.9928
 28.00     -2.8661      1.1139
 30.00     -3.1641      1.1641
 51.00     -2.5307      1.0279
 72.00     -2.3520      0.9919
 93.00     -2.3674      0.9975
114.00     -2.3799      1.0003
135.00     -2.3800      1.0002
156.00     -2.3792      1.0000
177.00     -2.3791      1.0000
198.00     -2.3792      1.0000
219.00     -2.3792      1.0000
240.00     -2.3792      1.0000
```


Swirling Flow over Disc in Axial Magnetic Field
with L = 60 and s = 0.24
Velocities
Radial Distance from Magnetic Field
axial velocity
tangential velocity
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
10
20
30
40
50
60


Swirling Flow over Disc in Axial Magnetic Field
with L = 120 and s = 0.144
Velocities
Radial Distance from Magnetic Field
axial velocity
tangential velocity
2
1
0
-1
-2
-3
-4
-5
0
20
40
60
80
100
120


Swirling Flow over Disc in Axial Magnetic Field
with L = 240 and s = 0.0864
Velocities
Radial Distance from Magnetic Field
axial velocity
tangential velocity
2
1
0
-1
-2
-3
-4
-5
-6
-7
0
50
100
150
200
250