NAG Toolbox

nag_wav_1d_mxolap_multi_fwd (c09dc)

1 Purpose

nag_wav_1d_mxolap_multi_fwd (c09dc) computes the one-dimensional multi-level maximal overlap discrete wavelet transform (MODWT). The initialization function nag_wav_1d_init (c09aa) must be called first to set up the MODWT options.

2 Syntax

```
[c, na, icomm, ifail] = nag_wav_1d_mxolap_multi_fwd(x, keepa, lenc, nwl, icomm, 'n', n)
[c, na, icomm, ifail] = c09dc(x, keepa, lenc, nwl, icomm, 'n', n)
```

3 Description

nag_wav_1d_mxolap_multi_fwd (c09dc) computes the multi-level MODWT for a data set, x_i, for $i = 1, 2, \ldots, n$, in one dimension. For a chosen number of levels, n_l, with $n_l \leq l_{\max}$, where $l_{\max}$ is returned by the initialization function nag_wav_1d_init (c09aa) in **nwlmax**, the transform is returned as a set of coefficients for the different levels stored in a single array. Periodic reflection is currently the only available end extension method to reduce the edge effects caused by finite data sets.

The argument **keepa** can be set to retain both approximation and detail coefficients at each level resulting in $n_l \times (n_a + n_d)$ coefficients being returned in the output array, **c**, where n_a is the number of approximation coefficients and n_d is the number of detail coefficients. Otherwise, only the detail coefficients are stored for each level along with the approximation coefficients for the final level, in which case the length of the output array, **c**, is $n_a + n_l \times n_d$. In the present implementation, for simplicity, n_a and n_d are chosen to be equal by adding zero padding to the wavelet filters where necessary.

4 References

Percival D B and Walden A T (2000) *Wavelet Methods for Time Series Analysis* Cambridge University Press

5 Parameters

5.1 Compulsory Input Parameters

1: **x**(**n**) – REAL (KIND=nag_wp) array

x contains the input dataset x_i, for $i = 1, 2, \ldots, n$.

2: **keepa** – CHARACTER(1)

Determines whether the approximation coefficients are stored in array **c** for every level of the computed transform or else only for the final level. In both cases, the detail coefficients are stored in **c** for every level computed.

keepa = 'A'
Retain approximation coefficients for all levels computed.

keepa = 'F'
Retain approximation coefficients for only the final level computed.

Constraint: **keepa** = 'A' or 'F'.

3: **lenc** – INTEGER

The dimension of the array **c**. **c** must be large enough to contain the number of wavelet coefficients.

If **keepa** = 'F', the total number of coefficients, n_c, is returned in **nwc** by the call to the initialization function nag_wav_1d_init (c09aa) and corresponds to the MODWT being continued for the maximum number of levels possible for the given data set. When the number of levels, n_l, is chosen to be less than the maximum, then the number of stored coefficients is correspondingly smaller and **lenc** can be reduced by noting that n_d detail coefficients are stored at each level, with the storage increased at the final level to contain the n_a approximation coefficients.

If **keepa** = 'A', n_d detail coefficients and n_a approximation coefficients are stored for each level computed, requiring **lenc** $\geq n_l \times (n_a + n_d) = 2 \times n_l \times n_a$, since the numbers of stored approximation and detail coefficients are equal. The number of approximation (or detail) coefficients at each level, n_a, is returned in **na**.

Constraints:

if **keepa** = 'F', **lenc** $\geq (n_l + 1) \times n_a$;
if **keepa** = 'A', **lenc** $\geq 2 \times n_l \times n_a$.

4: **nwl** – INTEGER

The number of levels, n_l, in the multi-level resolution to be performed.

Constraint: $1 \leq$ **nwl** $\leq l_{\max}$, where $l_{\max}$ is the value returned in **nwlmax** (the maximum number of levels) by the call to the initialization function nag_wav_1d_init (c09aa).

5: **icomm**(**100**) – INTEGER array

Contains details of the discrete wavelet transform and the problem dimension as setup in the call to the initialization function nag_wav_1d_init (c09aa).

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the dimension of the array **x**.

The number of elements, n, in the data array x.

Constraint: this must be the same as the value **n** passed to the initialization function nag_wav_1d_init (c09aa).

5.3 Output Parameters

1: **c**(**lenc**) – REAL (KIND=nag_wp) array

The coefficients of a multi-level wavelet transform of the dataset.

The coefficients are stored in **c** as follows:

If **keepa** = 'F',

c$(1 : n_a)$
Contains the level n_l approximation coefficients;

c$(n_a + (i-1) \times n_d + 1 : n_a + i \times n_d)$
Contains the level $(n_l - i + 1)$ detail coefficients, for $i = 1, 2, \ldots, n_l$;

If **keepa** = 'A',

c$((i-1) \times n_a + 1 : i \times n_a)$
Contains the level $(n_l - i + 1)$ approximation coefficients, for $i = 1, 2, \ldots, n_l$;

c$(n_l \times n_a + (i-1) \times n_d + 1 : n_l \times n_a + i \times n_d)$
Contains the level i detail coefficients, for $i = 1, 2, \ldots, n_l$;

The values n_a and n_d denote the numbers of approximation and detail coefficients respectively, which are equal and returned in **na**.

2: **na** – INTEGER

na contains the number of approximation coefficients, n_a, at each level which is equal to the number of detail coefficients, n_d. With periodic end extension (**mode** = 'P' in nag_wav_1d_init (c09aa)) this is the same as the length, **n**, of the data array, **x**.

3: **icomm**(**100**) – INTEGER array

Contains additional information on the computed transform.

4: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **n** is inconsistent with the value passed to the initialization function.

ifail = 2

On entry, **keepa** = ⟨*value*⟩ was an illegal value.

ifail = 4

lenc is too small.

ifail = 6

Constraint: **nwl** ≥ 1.

On entry, **nwl** is larger than the maximum number of levels returned by the initialization function.

ifail = 8

On entry, the initialization function nag_wav_1d_init (c09aa) has not been called first or it has not been called with **wtrans** = 'U', or the communication array **icomm** has become corrupted.

ifail = −99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = −399

Your licence key may have expired or may not have been installed correctly.

ifail = −999

Dynamic memory allocation failed.

7 Accuracy

The accuracy of the wavelet transform depends only on the floating-point operations used in the convolution and downsampling and should thus be close to ***machine precision***.

8 Further Comments

The wavelet coefficients at each level can be extracted from the output array **c** using the information contained in **na** on exit.

9 Example

A set of data values (**n** = 64) is decomposed using the MODWT over two levels and then the inverse (nag_wav_1d_mxolap_multi_inv (c09dd)) is applied to restore the original data set.

9.1 Program Text

```
function c09dc_example

fprintf('c09dc example results\n\n');

% Decompose x using maximal overlap discrete wavelet over 2 levels

n      = nag_int(64);
x      = [6.5271 6.5120 6.5016 6.5237 6.4625 6.3496 6.4025 6.4035 ...
          6.4407 6.4746 6.5095 6.6551 6.6100 6.5969 6.6083 6.6520 ...
          6.7113 6.7227 6.7196 6.7649 6.7794 6.8037 6.8308 6.7712 ...
          6.7067 6.7690 6.7068 6.7024 6.6463 6.6098 6.5900 6.5960 ...
          6.5457 6.5470 6.5797 6.5895 6.6275 6.6795 6.6598 6.6925 ...
          6.6873 6.7223 6.7205 6.6843 6.7030 6.6470 6.6008 6.6061 ...
          6.6097 6.6485 6.6394 6.6571 6.6357 6.6224 6.6073 6.6075 ...
          6.6379 6.6294 6.5906 6.6258 6.6369 6.6515 6.6826 6.7042];

wavnam = 'DB4';
mode   = 'Periodic';
wtrans = 'U';
keepa = 'All';
fprintf(' MLMODWT :: Wavelet : %10s, End mode : %10s, n = %10d\n',...
          wavnam, mode, n);
fprintf('         :: Keepa   : %10s\n\n',keepa);

% Setup for wavelet
[nwlmax, nf, nwc, icomm, ifail] = c09aa(wavnam, wtrans, mode, n);

% Compute decomposition over two levels
nwl = nag_int(2);
lenc = 2*n*nwl;
[c, na,  icomm, ifail] = c09dc(x, keepa, lenc, nwl, icomm);

fprintf(' Number of Levels                     : %10d\n',nwl);
fprintf(' Number of coefficients in each level : %10d\n\n',na);
fprintf(' Wavelet coefficients C : \n');
fprintf('%8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f\n',c)

% Reconstruct
[y, ifail] = c09dd(nwl, keepa, c, n, icomm);

fprintf('\n Reconstruction       Y : \n')
fprintf('%8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f\n',y)
```

9.2 Program Results

```
c09dc example results

 MLMODWT :: Wavelet :        DB4, End mode :   Periodic, n =         64
         :: Keepa   :        All

 Number of Levels                     :          2
 Number of coefficients in each level :         64

 Wavelet coefficients C :
  6.6448   6.6505   6.6415   6.6090   6.5631   6.5119   6.4657   6.4371
  6.4162   6.4041   6.4062   6.4235   6.4652   6.5191   6.5744   6.6170
```

```
  6.6375   6.6496   6.6575   6.6741   6.7038   6.7335   6.7633   6.7849
  6.7939   6.7970   6.7868   6.7649   6.7407   6.7102   6.6814   6.6571
  6.6269   6.5993   6.5773   6.5598   6.5574   6.5688   6.5881   6.6173
  6.6492   6.6741   6.6941   6.7052   6.7078   6.7083   6.7001   6.6842
  6.6616   6.6338   6.6146   6.6072   6.6139   6.6306   6.6428   6.6459
  6.6384   6.6252   6.6147   6.6113   6.6143   6.6189   6.6264   6.6361
  6.6719   6.5883   6.4958   6.4890   6.5103   6.4695   6.3900   6.3656
  6.4065   6.4444   6.4727   6.5273   6.6057   6.6409   6.6102   6.6001
  6.6469   6.7019   6.7288   6.7330   6.7501   6.7824   6.8064   6.8147
  6.7846   6.7332   6.7239   6.7297   6.6971   6.6508   6.6127   6.5897
  6.5818   6.5636   6.5476   6.5657   6.5980   6.6284   6.6627   6.6803
  6.6821   6.6941   6.7131   6.7182   6.7020   6.6824   6.6562   6.6140
  6.5942   6.6126   6.6378   6.6502   6.6498   6.6403   6.6233   6.6086
  6.6099   6.6260   6.6300   6.6112   6.6094   6.6358   6.6581   6.6778
  0.0107   0.0084   0.0003  -0.0065  -0.0000   0.0196   0.0191  -0.0152
 -0.0369  -0.0291  -0.0131   0.0227   0.0461   0.0005  -0.0488  -0.0145
  0.0518   0.0503  -0.0038  -0.0243  -0.0087  -0.0111  -0.0316  -0.0191
  0.0323   0.0461  -0.0001  -0.0300  -0.0107   0.0164   0.0112  -0.0156
 -0.0225  -0.0091   0.0090   0.0244   0.0050  -0.0281  -0.0150   0.0146
  0.0145   0.0034  -0.0019   0.0058   0.0188   0.0074  -0.0133  -0.0127
 -0.0062  -0.0008   0.0077   0.0022  -0.0151  -0.0192  -0.0041   0.0091
  0.0136   0.0230   0.0203  -0.0081  -0.0274  -0.0179  -0.0013   0.0074
 -0.0150   0.0126   0.0048  -0.0276  -0.0227   0.0639  -0.0184  -0.0048
 -0.0303   0.0180   0.0327  -0.0343   0.0119  -0.0046   0.0167   0.0025
 -0.0524   0.0369   0.0029   0.0055  -0.0070  -0.0134   0.0099   0.0088
 -0.0095   0.0103  -0.0114  -0.0181   0.0269   0.0132  -0.0371   0.0250
 -0.0186   0.0138   0.0022  -0.0058  -0.0112   0.0207  -0.0058  -0.0054
  0.0115  -0.0089  -0.0106   0.0180  -0.0096   0.0107  -0.0156   0.0068
  0.0074  -0.0242   0.0169   0.0075  -0.0045   0.0031  -0.0108   0.0092
 -0.0115   0.0061  -0.0002   0.0078  -0.0012  -0.0168   0.0074   0.0157

Reconstruction       Y :
  6.5271   6.5120   6.5016   6.5237   6.4625   6.3496   6.4025   6.4035
  6.4407   6.4746   6.5095   6.6551   6.6100   6.5969   6.6083   6.6520
  6.7113   6.7227   6.7196   6.7649   6.7794   6.8037   6.8308   6.7712
  6.7067   6.7690   6.7068   6.7024   6.6463   6.6098   6.5900   6.5960
  6.5457   6.5470   6.5797   6.5895   6.6275   6.6795   6.6598   6.6925
  6.6873   6.7223   6.7205   6.6843   6.7030   6.6470   6.6008   6.6061
  6.6097   6.6485   6.6394   6.6571   6.6357   6.6224   6.6073   6.6075
  6.6379   6.6294   6.5906   6.6258   6.6369   6.6515   6.6826   6.7042
```