# NAG Toolbox

# nag_zeros_quadratic_complex (c02ah)

## 1 Purpose

nag_zeros_quadratic_complex (c02ah) determines the roots of a quadratic equation with complex coefficients.

## 2 Syntax

```
[zsm, zlg, ifail] = nag_zeros_quadratic_complex(ar, ai, br, bi, cr, ci)
[zsm, zlg, ifail] = c02ah(ar, ai, br, bi, cr, ci)
```

## 3 Description

nag_zeros_quadratic_complex (c02ah) attempts to find the roots of the quadratic equation $az^2 + bz + c = 0$ (where $a$, $b$ and $c$ are complex coefficients), by carefully evaluating the 'standard' closed formula

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

It is based on the function CQDRTC from Smith (1967).

**Note:** it is not necessary to scale the coefficients prior to calling the function.

## 4 References

Smith B T (1967) ZERPOL: a zero finding algorithm for polynomials using Laguerre's method *Technical Report* Department of Computer Science, University of Toronto, Canada

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **ar** – REAL (KIND=nag_wp)
2: **ai** – REAL (KIND=nag_wp)

**ar** and **ai** must contain the real and imaginary parts respectively of $a$, the coefficient of $z^2$.

3: **br** – REAL (KIND=nag_wp)
4: **bi** – REAL (KIND=nag_wp)

**br** and **bi** must contain the real and imaginary parts respectively of $b$, the coefficient of $z$.

5: **cr** – REAL (KIND=nag_wp)
6: **ci** – REAL (KIND=nag_wp)

**cr** and **ci** must contain the real and imaginary parts respectively of $c$, the constant coefficient.

### 5.2 Optional Input Parameters

None.

### 5.3 Output Parameters

1: **zsm**(**2**) – REAL (KIND=nag_wp) array

The real and imaginary parts of the smallest root in magnitude are stored in **zsm**$(1)$ and **zsm**$(2)$ respectively.

2: **zlg**(**2**) – REAL (KIND=nag_wp) array

The real and imaginary parts of the largest root in magnitude are stored in **zlg**$(1)$ and **zlg**$(2)$ respectively.

3: **ifail** – INTEGER

**ifail** $= 0$ unless the function detects an error (see Section 5).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** $= 1$ (*warning*)

On entry, $(\mathbf{ar}, \mathbf{ai}) = (0, 0)$. In this case, **zsm**$(1)$ and **zsm**$(2)$ contain the real and imaginary parts respectively of the root $-c/b$.

**ifail** $= 2$

On entry, $(\mathbf{ar}, \mathbf{ai}) = (0, 0)$ and $(\mathbf{br}, \mathbf{bi}) = (0, 0)$. In this case, **zsm**$(1)$ contains the largest machine representable number (see nag_machine_real_largest (x02al)) and **zsm**$(2)$ contains zero.

**ifail** $= 3$

On entry, $(\mathbf{ar}, \mathbf{ai}) = (0, 0)$ and the root $-c/b$ overflows. In this case, **zsm**$(1)$ contains the largest machine representable number (see nag_machine_real_largest (x02al)) and **zsm**$(2)$ contains zero.

**ifail** $= 4$

On entry, $(\mathbf{cr}, \mathbf{ci}) = (0, 0)$ and the root $-b/a$ overflows. In this case, both **zsm**$(1)$ and **zsm**$(2)$ contain zero.

**ifail** $= 5$

On entry, $\tilde{b}$ is so large that $\tilde{b}^2$ is indistinguishable from $\tilde{b}^2 - 4\tilde{a}\tilde{c}$ and the root $-b/a$ overflows, where $\tilde{b} = \max(|\mathbf{br}|, |\mathbf{bi}|)$, $\tilde{a} = \max(|\mathbf{ar}|, |\mathbf{ai}|)$ and $\tilde{c} = \max(|\mathbf{cr}|, |\mathbf{ci}|)$. In this case, **zsm**$(1)$ and **zsm**$(2)$ contain the real and imaginary parts respectively of the root $-c/b$.

**ifail** $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

**ifail** $= -399$

Your licence key may have expired or may not have been installed correctly.

**ifail** $= -999$

Dynamic memory allocation failed.

If **ifail** $> 0$ on exit, then **zlg**$(1)$ contains the largest machine representable number (see nag_machine_real_largest (x02al)) and **zlg**$(2)$ contains zero.

## 7 Accuracy

If **ifail** $= 0$ on exit, then the computed roots should be accurate to within a small multiple of the ***machine precision*** except when underflow (or overflow) occurs, in which case the true roots are within a small multiple of the underflow (or overflow) threshold of the machine.

## 8 Further Comments

None.

## 9 Example

This example finds the roots of the quadratic equation $z^2 - (3.0 - 1.0i)z + (8.0 + 1.0i) = 0$.

### 9.1 Program Text

```
     function c02ah_example

fprintf('c02ah example results\n\n');

ar =  1;  ai = 0;
br = -3;  bi = 1;
cr =  8;  ci = 1;
[zsm, zlg, ifail] = c02ah(ar, ai, br, bi, cr, ci);

disp('Roots of the quadratic equation:');
z(1) = zsm(1) + i*zsm(2);
z(2) = zlg(1) + i*zlg(2);
disp(z');
```

### 9.2 Program Results

```
     c02ah example results

Roots of the quadratic equation:
   1.0000 - 2.0000i
   2.0000 + 3.0000i
```

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