

## NAG Toolbox

### nag\_mesh\_2d\_transform\_affine (d06da)

## 1 Purpose

nag\_mesh\_2d\_transform\_affine (d06da) is a utility which performs an affine transformation of a given mesh.

## 2 Syntax

```
[coori, edgeei, conni, cooro, edgeo, conno, ifail] = nag_mesh_2d_transform_affine
  (itype, trans, coori, edgeei, conni, itrace, 'nv', nv, 'nedge', nedge, 'nelt',
   nelt, 'ntrans', ntrans)
[coori, edgeei, conni, cooro, edgeo, conno, ifail] = d06da(itype, trans, coori,
  edgeei, conni, itrace, 'nv', nv, 'nedge', nedge, 'nelt', nelt, 'ntrans', ntrans)
```

## 3 Description

nag\_mesh\_2d\_transform\_affine (d06da) generates a mesh (coordinates, triangle/vertex connectivities and edge/vertex connectivities) resulting from an affine transformation of a given mesh. This transformation is of the form  $Y = A \times X + B$ , where

$Y$ ,  $X$  and  $B$  are in  $\mathbb{R}^2$ , and

$A$  is a real 2 by 2 matrix.

Such a transformation includes a translation, a rotation, a scale reduction or increase, a symmetric transformation with respect to a user-supplied line, a user-supplied analytic transformation, or a composition of several transformations.

This function is partly derived from material in the MODULEF package from INRIA (Institut National de Recherche en Informatique et Automatique).

## 4 References

None.

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **itype(ntrans)** – INTEGER array

**itype( $i$ )**, for  $i = 1, 2, \dots, ntrans$ , indicates the type of each transformation as follows:

**itype( $i$ ) = 0**  
Identity transformation.

**itype( $i$ ) = 1**  
Translation.

**itype( $i$ ) = 2**  
Symmetric transformation with respect to a user-supplied line.

**itype( $i$ ) = 3**  
Rotation.

**itype( $i$ ) = 4**  
Scaling.

**itype**( $i$ ) = 10

User-supplied analytic transformation.

Note that the transformations are applied in the order described in **itype**.

*Constraint:* **itype**( $i$ ) = 0, 1, 2, 3, 4 or 10, for  $i = 1, 2, \dots, \text{ntrans}$ .

- 2: **trans(6, ntrans)** – REAL (KIND=nag\_wp) array

The arguments for each transformation. For  $i = 1, 2, \dots, \text{ntrans}$ , **trans**(1,  $i$ ) to **trans**(6,  $i$ ) contain the arguments of the  $i$ th transformation.

If **itype**( $i$ ) = 0, elements **trans**(1,  $i$ ) to **trans**(6,  $i$ ) are not referenced.

If **itype**( $i$ ) = 1, the translation vector is  $\vec{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ , where  $a = \text{trans}(1, i)$  and  $b = \text{trans}(2, i)$ , while elements **trans**(3,  $i$ ) to **trans**(6,  $i$ ) are not referenced.

If **itype**( $i$ ) = 2, the user-supplied line is the curve  $\{(x, y) \in \mathbb{R}^2; \text{such that } ax + by + c = 0\}$ , where  $a = \text{trans}(1, i)$ ,  $b = \text{trans}(2, i)$  and  $c = \text{trans}(3, i)$ , while elements **trans**(4,  $i$ ) to **trans**(6,  $i$ ) are not referenced.

If **itype**( $i$ ) = 3, the centre of the rotation is  $(x_0, y_0)$  where  $x_0 = \text{trans}(1, i)$  and  $y_0 = \text{trans}(2, i)$ ,  $\theta = \text{trans}(3, i)$  is its angle in degrees, while elements **trans**(4,  $i$ ) to **trans**(6,  $i$ ) are not referenced.

If **itype**( $i$ ) = 4,  $a = \text{trans}(1, i)$  is the scaling coefficient in the  $x$ -direction,  $b = \text{trans}(2, i)$  is the scaling coefficient in the  $y$ -direction, and  $(x_0, y_0)$  are the scaling centre coordinates, with  $x_0 = \text{trans}(3, i)$  and  $y_0 = \text{trans}(4, i)$ ; while elements **trans**(5,  $i$ ) to **trans**(6,  $i$ ) are not referenced.

If **itype**( $i$ ) = 10, the user-supplied analytic affine transformation  $Y = A \times X + B$  is such that  $A = (a_{kl})_{1 \leq k, l \leq 2}$  and  $B = (b_k)_{1 \leq k \leq 2}$  where  $a_{kl} = \text{trans}(2 \times (k - 1) + l, i)$ , and  $b_k = \text{trans}(4 + k, i)$  with  $k, l = 1, 2$ .

- 3: **coori(2, nv)** – REAL (KIND=nag\_wp) array

**coori**(1,  $i$ ) contains the  $x$  coordinate of the  $i$ th vertex of the input mesh, for  $i = 1, 2, \dots, \text{nv}$ ; while **coori**(2,  $i$ ) contains the corresponding  $y$  coordinate.

- 4: **edgei(3, nedge)** – INTEGER array

The specification of the boundary or interface edges. **edgei**(1,  $j$ ) and **edgei**(2,  $j$ ) contain the vertex numbers of the two end points of the  $j$ th boundary edge. **edgei**(3,  $j$ ) is a user-supplied tag for the  $j$ th boundary edge.

*Constraint:*  $1 \leq \text{edgei}(i, j) \leq \text{nv}$  and  $\text{edgei}(1, j) \neq \text{edgei}(2, j)$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, \text{nedge}$ .

- 5: **conni(3, nelts)** – INTEGER array

The connectivity of the input mesh between triangles and vertices. For each triangle  $j$ , **conni**( $i, j$ ) gives the indices of its three vertices (in anticlockwise order), for  $i = 1, 2, 3$  and  $j = 1, 2, \dots, \text{nelts}$ .

*Constraints:*

$$1 \leq \text{conni}(i, j) \leq \text{nv};$$

$$\text{conni}(1, j) \neq \text{conni}(2, j);$$

$$\text{conni}(1, j) \neq \text{conni}(3, j) \text{ and } \text{conni}(2, j) \neq \text{conni}(3, j), \text{ for } i = 1, 2, 3 \text{ and } j = 1, 2, \dots, \text{nelts}.$$

- 6: **itrace** – INTEGER

The level of trace information required from nag\_mesh\_2d\_transform\_affine (d06da).

**itrace**  $\leq 0$

No output is generated.

**itrace**  $\geq 1$

Details of each transformation, the matrix  $A$  and the vector  $B$  of the final transformation, which is the composition of all the **ntrans** transformations, are printed on the current advisory message unit (see nag\_file\_set\_unit\_advisory (x04ab)).

## 5.2 Optional Input Parameters

1: **nv** – INTEGER

*Default:* the dimension of the array **coori**.

The total number of vertices in the input mesh.

*Constraint:*  $\mathbf{nv} \geq 3$ .

2: **nedge** – INTEGER

*Default:* the dimension of the array **edgei**.

The number of the boundary or interface edges in the input mesh.

*Constraint:*  $\mathbf{nedge} \geq 1$ .

3: **nelt** – INTEGER

*Default:* the dimension of the array **conni**.

The number of triangles in the input mesh.

*Constraint:*  $\mathbf{nelt} \leq 2 \times \mathbf{nv} - 1$ .

4: **ntrans** – INTEGER

*Default:* the dimension of the arrays **itype**, **trans**. (An error is raised if these dimensions are not equal.)

The number of transformations of the input mesh.

*Constraint:*  $\mathbf{ntrans} \geq 1$ .

## 5.3 Output Parameters

1: **coori(2, nv)** – REAL (KIND=nag\_wp) array

see Section 9.

2: **edgei(3, nedge)** – INTEGER array

See Section 9.

3: **conni(3, nelt)** – INTEGER array

See Section 9.

4: **cooro(2, nv)** – REAL (KIND=nag\_wp) array

**cooro**(1,  $i$ ) will contain the  $x$  coordinate of the  $i$ th vertex of the transformed mesh, for  $i = 1, 2, \dots, \mathbf{nv}$ ; while **cooro**(2,  $i$ ) will contain the corresponding  $y$  coordinate.

5: **edgeo(3, nedge)** – INTEGER array

The specification of the boundary or interface edges of the transformed mesh. If the number of symmetric transformations is even or zero then **edgeo**( $i, j$ ) = **edgei**( $i, j$ ), for  $i = 1, 2, 3$  and  $j = 1, 2, \dots, \mathbf{nedge}$ ; otherwise **edgeo**(1,  $j$ ) = **edgei**(2,  $j$ ), **edgeo**(2,  $j$ ) = **edgei**(1,  $j$ ) and **edgeo**(3,  $j$ ) = **edgei**(3,  $j$ ), for  $j = 1, 2, \dots, \mathbf{nedge}$ .

6: **conno(3, nelt)** – INTEGER array

The connectivity of the transformed mesh between triangles and vertices. If the number of symmetric transformations is even or zero then  $\text{conno}(i,j) = \text{conni}(i,j)$ , for  $i = 1, 2, 3$  and  $j = 1, 2, \dots, \text{nelt}$ ; otherwise  $\text{conno}(1,j) = \text{conni}(1,j)$ ,  $\text{conno}(2,j) = \text{conni}(3,j)$  and  $\text{conno}(3,j) = \text{conni}(2,j)$ , for  $j = 1, 2, \dots, \text{nelt}$ .

7: **ifail** – INTEGER

**ifail** = 0 unless the function detects an error (see Section 5).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** = 1

On entry,  $\text{nv} < 3$ ;  
 or  $\text{nelt} > 2 \times \text{nv} - 1$ ;  
 or  $\text{nedge} < 1$ ;  
 or  $\text{edgei}(i,j) < 1$  or  $\text{edgei}(i,j) > \text{nv}$  for some  $i = 1, 2$  and  $j = 1, 2, \dots, \text{nedge}$ ;  
 or  $\text{edgei}(1,j) = \text{edgei}(2,j)$  for some  $j = 1, 2, \dots, \text{nedge}$ ;  
 or  $\text{conni}(i,j) < 1$  or  $\text{conni}(i,j) > \text{nv}$  for some  $i = 1, 2, 3$  and  $j = 1, 2, \dots, \text{nelt}$ ;  
 or  $\text{conni}(1,j) = \text{conni}(2,j)$  or  $\text{conni}(1,j) = \text{conni}(3,j)$  or  
 $\text{conni}(2,j) = \text{conni}(3,j)$  for some  $j = 1, 2, \dots, \text{nelt}$ ;  
 or  $\text{ntrans} < 1$ ;  
 or  $\text{itype}(i) \neq 0, 1, 2, 3, 4$  or 10 for some  $i = 1, 2, \dots, \text{ntrans}$ ;  
 or  $\text{lrwork} < 12 \times \text{ntrans}$ .

**ifail** = 2

A serious error has occurred in an internal call to an auxiliary function. Check the input mesh especially the triangles/vertices and the edges/vertices connectivities as well as the details of each transformations.

**ifail** = -99

An unexpected error has been triggered by this routine. Please contact NAG.

**ifail** = -399

Your licence key may have expired or may not have been installed correctly.

**ifail** = -999

Dynamic memory allocation failed.

## 7 Accuracy

Not applicable.

## 8 Further Comments

You may not wish to save the input mesh (**coori**, **edgei** and **conni**) and could call `nag_mesh_2d_transform_affine` (d06da) using the same arguments for the input and the output (transformed) mesh.

## 9 Example

For an example of the use of this utility function, see Section 10 in `nag_mesh_2d_join` (d06db).

## 9.1 Program Text

```

function d06da_example

fprintf('d06da example results\n\n');

coor1 = zeros(2,400);
for j = 1:20
    for i = 1:20
        coor1(1,(j-1)*20+i) = (i-1)/19;
        coor1(2,(j-1)*20+i) = (j-1)/19;
    end
end
edge1 = ones(3, 76, nag_int_name);
edge1(1, 1:76) = nag_int([1:19,20*(1:19),401-(1:19),401-20*(1:19)]);
edge1(2, 1:76) = nag_int([2:19,20*(1:19),401-(1:19),401-20*(1:19),1]);
conn1 = zeros(3, 722, nag_int_name);
ind = -1;
for i=1:379
    if (rem(i, 20) ~= 0)
        ind = ind+2;
    conn1(1, ind) = nag_int(i);
    conn1(1, ind+1) = nag_int(i);
    conn1(2, ind) = nag_int(i+1);
    conn1(2, ind+1) = nag_int(i+21);
    conn1(3, ind) = nag_int(i+21);
    conn1(3, ind+1) = nag_int(i+20);
    end
end
reft1 = ones(722, 1, nag_int_name);
reft2 = reft1;
reft2(:) = nag_int(2);
itype = [nag_int(1)];
itrace = nag_int(1);

% Transform the first domain to obtain an overlapping second domain
trans = [15/19; 17/19; 0; 0; 0];
[coor1, edge1, conn1, coor2, edge2, conn2, ifail] = ...
d06da( ...
    itype, trans, coor1, edge1, conn1, itrace);

% Restitch the meshes
[nv3, nelt3, nedge3, coor3, edge3, conn3, reft3, ifail] = ...
d06db( ...
    coor1, edge1, conn1, reft1, coor2, edge2, conn2, reft2, itrace);

% Plot the result
fig1 = figure;
triplot(transpose(double(conn3(:,1:nelt3))), coor3(1,:), coor3(2,:));
title ('Interior mesh of the two overlapping squares geometry');

% Now consider a partitioned second domain
trans = [1; 0; 0; 0; 0];
[coor1, edge1, conn1, coor2, edge2, conn2, ifail] = ...
d06da( ...
    itype, trans, coor1, edge1, conn1, itrace);

% Restitch the meshes
[nv3, nelt3, nedge3, coor3, edge3, conn3, reft3, ifail] = ...
d06db( ...
    coor1, edge1, conn1, reft1, coor2, edge2, conn2, reft2, itrace);

% Plot the result
fig2 = figure;
triplot(transpose(double(conn3(:,1:nelt3))), coor3(1,:), coor3(2,:));
title ('Interior mesh of the two partitioned squares geometry');

```

## 9.2 Program Results

d06da example results

```
Transformation 1: translation
  translation vector:  0.7895      0.8947

Final transformation matrix y = A*x + b:
  1.000      0.000      0.7895
  0.000      1.000      0.8947

Transformation 1: translation
  translation vector:  1.000      0.000

Final transformation matrix y = A*x + b:
  1.000      0.000      1.000
  0.000      1.000      0.000
```



