Chapter 6

Eigenvalue and Least-squares Problems

1 Scope of the Chapter

This chapter provides procedures for various types of matrix eigenvalue problems, including singular value problems.

It also provides procedures for solving linear least-squares problems, using the singular value decomposition or QR factorization.

All the procedures in this chapter are generic procedures which can handle either real or complex data.

A common theme in many of the algorithms of this chapter is the use of real orthogonal or complex unitary matrices to transform problems into simpler forms while leaving essential properties invariant (especially the 2-norms of vectors and matrices).

2 Available Modules

Module 6.1: `nag_sym_eig` — **Standard symmetric eigenvalue problems**

Provides procedures for:

- solving the standard symmetric eigenvalue problem $Az = \lambda z$, where the matrix A is *real symmetric* or *complex Hermitian*;
- performing various computational sub-tasks involved in solving such problems. (These are lower-level procedures, intended for more experienced users.)

Module 6.2: `nag_nsym_eig` — **Standard nonsymmetric eigenvalue problems**

Provides procedures for:

- solving the standard nonsymmetric eigenvalue problem $Ax = \lambda x$, where A is a *general* real or complex square matrix;
- computing the related Schur factorization of A.

Module 6.3: `nag_svd` — **Singular value decomposition (SVD)**

Provides procedures for:

- computing the singular value decomposition of a real or complex matrix;
- performing various computational sub-tasks involved in these problems. (These are lower-level procedures, intended for more experienced users.)

Module 6.4: `nag_lin_lsq` — **Linear least-squares problems**

Provides procedures for:

- solving a linear least-squares problem (which may be rank-deficient), by computing the SVD of the coefficient matrix;
- solving a linear least-squares problem, assuming that the SVD of the coefficient matrix has already been computed;
- computing the QR factorization of a real or complex matrix and performing related computational tasks;
- solving a linear least-squares problem, assuming that the QR factorization of the coefficient matrix has already been computed; determination of the rank of the problem is either left as the responsibility of the user, or is handled by computing the SVD of R.

Module 6.5: `nag_sym_gen_eig` — **Symmetric-definite generalized eigenvalue problems**

Provides procedures for:

- solving the generalized eigenvalue problem $Az = \lambda Bz$, where A and B are *real symmetric* or *complex Hermitian*, and B is *positive definite*.

Module 6.6: `nag_nsym_gen_eig` — **Nonsymmetric generalized eigenvalue problems**

Provides procedures for:

- solving the generalized nonsymmetric eigenvalue problem $Ax = \lambda Bx$, where A and B are *general* real or complex square matrices;
- computing the related generalized Schur factorization of the matrix pencil $A - \lambda B$.