NAG Library Routine Document

G01GBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1   Purpose

G01GBF returns the lower tail probability for the noncentral Student's t-distribution, via the routine name.

2   Specification

```
FUNCTION G01GBF (T, DF, DELTA, TOL, MAXIT, IFAIL)
REAL (KIND=nag_wp) G01GBF

INTEGER           MAXIT, IFAIL
REAL (KIND=nag_wp) T, DF, DELTA, TOL
```

3   Description

The lower tail probability of the noncentral Student's t-distribution with ν degrees of freedom and noncentrality parameter δ, $P(T \leq t : \nu;\delta)$, is defined by

$$P(T \leq t : \nu;\delta) = C_\nu \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha u - \delta} e^{-x^2/2}\, dx \right) u^{\nu-1} e^{-u^2/2} du, \quad \nu > 0.0$$

with

$$C_\nu = \frac{1}{\Gamma\left(\frac{1}{2}\nu\right) 2^{(\nu-2)/2}}, \quad \alpha = \frac{t}{\sqrt{\nu}}.$$

The probability is computed in one of two ways.

(i)   When $t = 0.0$, the relationship to the normal is used:

$$P(T \leq t : \nu;\delta) = \frac{1}{\sqrt{2\pi}} \int_\delta^\infty e^{-u^2/2}\, du.$$

(ii)   Otherwise the series expansion described in Equation 9 of Amos (1964) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4   References

Amos D E (1964) Representations of the central and non-central t-distributions *Biometrika* **51** 451–458

5   Arguments

1:   T – REAL (KIND=nag_wp)   *Input*

On entry: t, the deviate from the Student's t-distribution with ν degrees of freedom.

2:   DF – REAL (KIND=nag_wp)   *Input*

On entry: ν, the degrees of freedom of the Student's t-distribution.

Constraint: DF $\geq$ 1.0.

3:     DELTA – REAL (KIND=nag_wp)   *Input*

      On entry: δ, the noncentrality argument of the Students t-distribution.

4:     TOL – REAL (KIND=nag_wp)   *Input*

      On entry: the absolute accuracy required by you in the results. If G01GBF is entered with TOL greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see X02AJF), then the value of $10 \times$ ***machine precision*** is used instead.

5:     MAXIT – INTEGER   *Input*

      On entry: the maximum number of terms that are used in each of the summations.

      Suggested value: 100. See Section 9 for further comments.

      Constraint: MAXIT ≥ 1.

6:     IFAIL – INTEGER   *Input/Output*

      On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

      For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

      On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6   Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If on exit IFAIL $\neq 0$, then G01GBF returns 0.0.

IFAIL $= 1$

      On entry, DF < 1.0.

IFAIL $= 2$

      On entry, MAXIT < 1.

IFAIL $= 3$

      One of the series has failed to converge. Reconsider the requested tolerance and/or maximum number of iterations.

IFAIL $= 4$

      The probability is too small to calculate accurately.

IFAIL $= -99$

      An unexpected error has been triggered by this routine. Please contact NAG.

      See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -399$

> Your licence key may have expired or may not have been installed correctly.

> See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -999$

> Dynamic memory allocation failed.

> See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than TOL. There may also be some loss of accuracy due to calculation of gamma functions.

8 Parallelism and Performance

G01GBF is not threaded in any implementation.

9 Further Comments

The rate of convergence of the series depends, in part, on the quantity $t^2/(t^2 + \nu)$. The smaller this quantity the faster the convergence. Thus for large t and small ν the convergence may be slow. If ν is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the t-distribution to the F-distribution can be used:

$$F = T^2, \lambda = \delta^2, \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,$$

and a call made to G01GDF.

Note that G01GBF only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

10 Example

This example reads values from, and degrees of freedom for, and noncentrality arguments of the noncentral Student's t-distributions, calculates the lower tail probabilities and prints all these values until the end of data is reached.

10.1 Program Text

```
    Program g01gbfe

!     G01GBF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: g01gbf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter                :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)                :: delta, df, prob, t, tol
      Integer                           :: ifail, maxit
!     .. Executable Statements ..
      Write (nout,*) 'G01GBF Example Program Results'
      Write (nout,*)
```

```
!       Skip heading in data file
        Read (nin,*)

!       Display titles
        Write (nout,*) '     T       DF    DELTA    PROB'
        Write (nout,*)

!       Use default tolerance and suggested number of iterations
        tol = 0.0E0_nag_wp
        maxit = 100

d_lp: Do
          Read (nin,*,Iostat=ifail) t, df, delta
          If (ifail/=0) Then
            Exit d_lp
          End If

!         Calculate probability
          ifail = 0
          prob = g01gbf(t,df,delta,tol,maxit,ifail)

!         Display results
          Write (nout,99999) t, df, delta, prob
        End Do d_lp

99999 Format (1X,3F8.3,F8.4)
      End Program g01gbfe
```

10.2 Program Data

```
G01GBF Example Program Data
 -1.528  20.0  2.0               :T DF DELTA
 -0.188   7.5  1.0               :T DF DELTA
  1.138  45.0  0.0               :T DF DELTA
```

10.3 Program Results

```
 G01GBF Example Program Results

      T       DF    DELTA    PROB

  -1.528  20.000   2.000  0.0003
  -0.188   7.500   1.000  0.1189
   1.138  45.000   0.000  0.8694
```