

## NAG Library Routine Document

### F08WSF (ZGGHRD)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

#### 1 Purpose

F08WSF (ZGGHRD) reduces a pair of complex matrices  $(A, B)$ , where  $B$  is upper triangular, to the generalized upper Hessenberg form using unitary transformations.

#### 2 Specification

SUBROUTINE F08WSF (COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z, &  
LDZ, INFO)

INTEGER N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO  
COMPLEX (KIND=nag\_wp) A(LDA,\*), B(LDB,\*), Q(LDQ,\*), Z(LDZ,\*)  
CHARACTER(1) COMPQ, COMPZ

The routine may be called by its LAPACK name *zgghrd*.

#### 3 Description

F08WSF (ZGGHRD) is usually the third step in the solution of the complex generalized eigenvalue problem

$$Ax = \lambda Bx.$$

The (optional) first step balances the two matrices using F08WVF (ZGGBAL). In the second step, matrix  $B$  is reduced to upper triangular form using the  $QR$  factorization routine F08ASF (ZGEQRF) and this unitary transformation  $Q$  is applied to matrix  $A$  by calling F08AUF (ZUNMQR).

F08WSF (ZGGHRD) reduces a pair of complex matrices  $(A, B)$ , where  $B$  is triangular, to the generalized upper Hessenberg form using unitary transformations. This two-sided transformation is of the form

$$\begin{aligned} Q^H A Z &= H \\ Q^H B Z &= T \end{aligned}$$

where  $H$  is an upper Hessenberg matrix,  $T$  is an upper triangular matrix and  $Q$  and  $Z$  are unitary matrices determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices  $Q_1$  and  $Z_1$ , so that

$$\begin{aligned} Q_1 A Z_1^H &= (Q_1 Q) H (Z_1 Z)^H, \\ Q_1 B Z_1^H &= (Q_1 Q) T (Z_1 Z)^H. \end{aligned}$$

#### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Moler C B and Stewart G W (1973) An algorithm for generalized matrix eigenproblems *SIAM J. Numer. Anal.* **10** 241–256

## 5 Arguments

- 1: COMPQ – CHARACTER(1) *Input*  
*On entry:* specifies the form of the computed unitary matrix  $Q$ .  
 COMPQ = 'N'  
     Do not compute  $Q$ .  
 COMPQ = 'I'  
     The unitary matrix  $Q$  is returned.  
 COMPQ = 'V'  
      $Q$  must contain a unitary matrix  $Q_1$ , and the product  $Q_1Q$  is returned.  
*Constraint:* COMPQ = 'N', 'I' or 'V'.
- 2: COMPZ – CHARACTER(1) *Input*  
*On entry:* specifies the form of the computed unitary matrix  $Z$ .  
 COMPZ = 'N'  
     Do not compute  $Z$ .  
 COMPZ = 'V'  
      $Z$  must contain a unitary matrix  $Z_1$ , and the product  $Z_1Z$  is returned.  
 COMPZ = 'I'  
     The unitary matrix  $Z$  is returned.  
*Constraint:* COMPZ = 'N', 'V' or 'I'.
- 3: N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrices  $A$  and  $B$ .  
*Constraint:*  $N \geq 0$ .
- 4: ILO – INTEGER *Input*  
 5: IHI – INTEGER *Input*  
*On entry:*  $i_{lo}$  and  $i_{hi}$  as determined by a previous call to F08WVF (ZGGBAL). Otherwise, they should be set to 1 and  $n$ , respectively.  
*Constraints:*  
     if  $N > 0$ ,  $1 \leq ILO \leq IHI \leq N$ ;  
     if  $N = 0$ ,  $ILO = 1$  and  $IHI = 0$ .
- 6: A(LDA,\*) – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array  $A$  must be at least  $\max(1, N)$ .  
*On entry:* the matrix  $A$  of the matrix pair  $(A, B)$ . Usually, this is the matrix  $A$  returned by F08AUF (ZUNMQR).  
*On exit:*  $A$  is overwritten by the upper Hessenberg matrix  $H$ .
- 7: LDA – INTEGER *Input*  
*On entry:* the first dimension of the array  $A$  as declared in the (sub)program from which F08WSF (ZGGHRD) is called.  
*Constraint:*  $LDA \geq \max(1, N)$ .

- 8: B(LDB,\*) – COMPLEX (KIND=nag\_wp) array Input/Output  
**Note:** the second dimension of the array B must be at least  $\max(1, N)$ .  
*On entry:* the upper triangular matrix  $B$  of the matrix pair  $(A, B)$ . Usually, this is the matrix  $B$  returned by the  $QR$  factorization routine F08ASF (ZGEQRF).  
*On exit:* B is overwritten by the upper triangular matrix  $T$ .
- 9: LDB – INTEGER Input  
*On entry:* the first dimension of the array B as declared in the (sub)program from which F08WSF (ZGGHRD) is called.  
*Constraint:*  $LDB \geq \max(1, N)$ .
- 10: Q(LDQ,\*) – COMPLEX (KIND=nag\_wp) array Input/Output  
**Note:** the second dimension of the array Q must be at least  $\max(1, N)$  if COMPQ = 'I' or 'V' and at least 1 if COMPQ = 'N'.  
*On entry:* if COMPQ = 'V', Q must contain a unitary matrix  $Q_1$ .  
If COMPQ = 'N', Q is not referenced.  
*On exit:* if COMPQ = 'I', Q contains the unitary matrix  $Q$ .  
If COMPQ = 'V', Q is overwritten by  $Q_1Q$ .
- 11: LDQ – INTEGER Input  
*On entry:* the first dimension of the array Q as declared in the (sub)program from which F08WSF (ZGGHRD) is called.  
*Constraints:*  
if COMPQ = 'I' or 'V',  $LDQ \geq \max(1, N)$ ;  
if COMPQ = 'N',  $LDQ \geq 1$ .
- 12: Z(LDZ,\*) – COMPLEX (KIND=nag\_wp) array Input/Output  
**Note:** the second dimension of the array Z must be at least  $\max(1, N)$  if COMPZ = 'V' or 'I' and at least 1 if COMPZ = 'N'.  
*On entry:* if COMPZ = 'V', Z must contain a unitary matrix  $Z_1$ .  
If COMPZ = 'N', Z is not referenced.  
*On exit:* if COMPZ = 'I', Z contains the unitary matrix  $Z$ .  
If COMPZ = 'V', Z is overwritten by  $Z_1Z$ .
- 13: LDZ – INTEGER Input  
*On entry:* the first dimension of the array Z as declared in the (sub)program from which F08WSF (ZGGHRD) is called.  
*Constraints:*  
if COMPZ = 'V' or 'I',  $LDZ \geq \max(1, N)$ ;  
if COMPZ = 'N',  $LDZ \geq 1$ .
- 14: INFO – INTEGER Output  
*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

INFO < 0

If INFO =  $-i$ , argument  $i$  had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The reduction to the generalized Hessenberg form is implemented using unitary transformations which are backward stable.

## 8 Parallelism and Performance

F08WSF (ZGGHRD) is not threaded in any implementation.

## 9 Further Comments

This routine is usually followed by F08XSF (ZHGEQZ) which implements the  $QZ$  algorithm for computing generalized eigenvalues of a reduced pair of matrices.

The real analogue of this routine is F08WEF (DGGHRD).

## 10 Example

See Section 10 in F08XSF (ZHGEQZ) and F08YXF (ZTGEVC).

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