NAG Library Routine Document

S18CRF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1   Purpose

S18CRF returns an array of values of the scaled modified Bessel function $e^x K_1(x)$.

2   Specification

```
SUBROUTINE S18CRF (N, X, F, IVALID, IFAIL)

INTEGER           N, IVALID(N), IFAIL
REAL (KIND=nag_wp) X(N), F(N)
```

3   Description

S18CRF evaluates an approximation to $e^{x_i} K_1(x_i)$, where K_1 is a modified Bessel function of the second kind for an array of arguments x_i, for $i = 1, 2, \ldots, n$. The scaling factor e^x removes most of the variation in $K_1(x)$.

The routine uses the same Chebyshev expansions as S18ARF, which returns an array of the unscaled values of $K_1(x)$.

4   References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5   Arguments

1:   N – INTEGER   *Input*

On entry: n, the number of points.

Constraint: $N \geq 0$.

2:   X(N) – REAL (KIND=nag_wp) array   *Input*

On entry: the argument x_i of the function, for $i = 1, 2, \ldots, N$.

Constraint: $X(i) > 0.0$, for $i = 1, 2, \ldots, N$.

3:   F(N) – REAL (KIND=nag_wp) array   *Output*

On exit: $e^{x_i} K_1(x_i)$, the function values.

4:   IVALID(N) – INTEGER array   *Output*

On exit: IVALID(i) contains the error code for x_i, for $i = 1, 2, \ldots, N$.

IVALID(i) = 0
  No error.

IVALID(i) = 1

  On entry, $x_i \leq 0.0$, $K_1(x_i)$ is undefined. F(i) contains 0.0.

IVALID$(i) = 2$

x_i is too close to zero, as determined by the value of the safe-range parameter X02AMF: there is a danger of causing overflow. F(i) contains the reciprocal of the safe-range parameter.

5:    IFAIL – INTEGER   *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6   Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, at least one value of X was invalid.
Check IVALID for more information.

IFAIL $= 2$

On entry, N $= \langle value \rangle$.
Constraint: N ≥ 0.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7   Accuracy

Relative errors in the argument are attenuated when propagated into the function value. When the accuracy of the argument is essentially limited by the *machine precision*, the accuracy of the function value will be similarly limited by at most a small multiple of the *machine precision*.

8   Parallelism and Performance

S18CRF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of X from a file, evaluates the function at each value of x_i and prints the results.

10.1 Program Text

```
    Program s18crfe

!     S18CRF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s18crf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter              :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                         :: i, ifail, n
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: f(:), x(:)
      Integer, Allocatable            :: ivalid(:)
!     .. Executable Statements ..
      Write (nout,*) 'S18CRF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '      X            F           IVALID'
      Write (nout,*)

      Read (nin,*) n

      Allocate (x(n),f(n),ivalid(n))

      Read (nin,*) x(1:n)

      ifail = 0
      Call s18crf(n,x,f,ivalid,ifail)

      Do i = 1, n
        Write (nout,99999) x(i), f(i), ivalid(i)
      End Do

99999 Format (1X,1P,2E12.3,I5)
      End Program s18crfe
```

10.2 Program Data

```
S18CRF Example Program Data

6

0.4 0.6 1.4 2.5 10.0 1000.0
```

10.3 Program Results

```
S18CRF Example Program Results

      X            F          IVALID

   4.000E-01    3.259E+00        0
   6.000E-01    2.374E+00        0
   1.400E+00    1.301E+00        0
   2.500E+00    9.002E-01        0
   1.000E+01    4.108E-01        0
   1.000E+03    3.965E-02        0
```