NAG Library Routine Document

S14AGF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S14AGF returns the value of the logarithm of the gamma function $\ln \Gamma(z)$ for complex z, via the function name.

2 Specification

```
FUNCTION S14AGF (Z, IFAIL)
COMPLEX (KIND=nag_wp) S14AGF
INTEGER               IFAIL
COMPLEX (KIND=nag_wp) Z
```

3 Description

S14AGF evaluates an approximation to the logarithm of the gamma function $\ln \Gamma(z)$ defined for $\mathrm{Re}(z) > 0$ by

$$\ln \Gamma(z) = \ln \int_0^\infty e^{-t} t^{z-1} dt$$

where $z = x + iy$ is complex. It is extended to the rest of the complex plane by analytic continuation unless $y = 0$, in which case z is real and each of the points $z = 0, -1, -2, \ldots$ is a singularity and a branch point.

S14AGF is based on the method proposed by KÎlbig (1972) in which the value of $\ln \Gamma(z)$ is computed in the different regions of the z plane by means of the formulae

$$\begin{aligned}
\ln \Gamma(z) &= \left(z - \tfrac{1}{2}\right) \ln z - z + \tfrac{1}{2} \ln 2\pi + z \sum_{k=1}^{K} \frac{B_{2k}}{2k(2k-1)} z^{-2k} + R_K(z) && \text{if } x \geq x_0 \geq 0, \\
&= \ln \Gamma(z+n) - \ln \prod_{\nu=0}^{n-1} (z+\nu) && \text{if } x_0 > x \geq 0, \\
&= \ln \pi - \ln \Gamma(1-z) - \ln(\sin \pi z) && \text{if } x < 0,
\end{aligned}$$

where $n = [x_0] - [x]$, $\{B_{2k}\}$ are Bernoulli numbers (see Abramowitz and Stegun (1972)) and $[x]$ is the largest integer $\leq x$. Note that care is taken to ensure that the imaginary part is computed correctly, and not merely modulo 2π.

The routine uses the values $K = 10$ and $x_0 = 7$. The remainder term $R_K(z)$ is discussed in Section 7.

To obtain the value of $\ln \Gamma(z)$ when z is real and positive, S14ABF can be used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

KÎlbig K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

5 Arguments

1: Z – COMPLEX (KIND=nag_wp) *Input*

On entry: the argument z of the function.

Constraint: Z must not be 'too close' (see Section 6) to a non-positive integer when Z = 0.0.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, Re(Z) is 'too close' to a non-positive integer when Im(Z) $= 0.0$. That is, abs(Re(Z) – nint(Re(Z))) < ***machine precision*** × nint(abs(Re(Z))).

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The remainder term $R_K(z)$ satisfies the following error bound:

$$\begin{aligned} |R_K(z)| &\leq \frac{|B_{2K}|}{|(2K-1)|} z^{1-2K} \\ &\leq \frac{|B_{2K}|}{|(2K-1)|} x^{1-2K} \text{if } x \geq 0. \end{aligned}$$

Thus $|R_{10}(7)| < 2.5 \times 10^{-15}$ and hence in theory the routine is capable of achieving an accuracy of approximately 15 significant digits.

8 Parallelism and Performance

S14AGF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example evaluates the logarithm of the gamma function $\ln \Gamma(z)$ at $z = -1.5 + 2.5i$, and prints the results.

10.1 Program Text

```
    Program s14agfe

!     S14AGF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s14agf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Complex (Kind=nag_wp)            :: y, z
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S14AGF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '       Z                 ln(Gamma(Z))'
      Write (nout,*)

data: Do
        Read (nin,*,Iostat=ioerr) z

        If (ioerr<0) Then
          Exit data
        End If

        ifail = -1
        y = s14agf(z,ifail)

        If (ifail<0) Then
          Exit data
        End If

        Write (nout,99999) z, y
      End Do data

99999 Format (1X,'(',F5.1,',',F5.1,' )   (',1P,E12.4,',',E12.4,' )')
    End Program s14agfe
```

10.2 Program Data

```
S14AGF Example Program Data
 (-1.5, 2.5)  : Value of Z
```

10.3 Program Results

```
 S14AGF Example Program Results

        Z                  ln(Gamma(Z))

 ( -1.5,  2.5 )   ( -5.0140E+00, -4.0718E+00 )
```